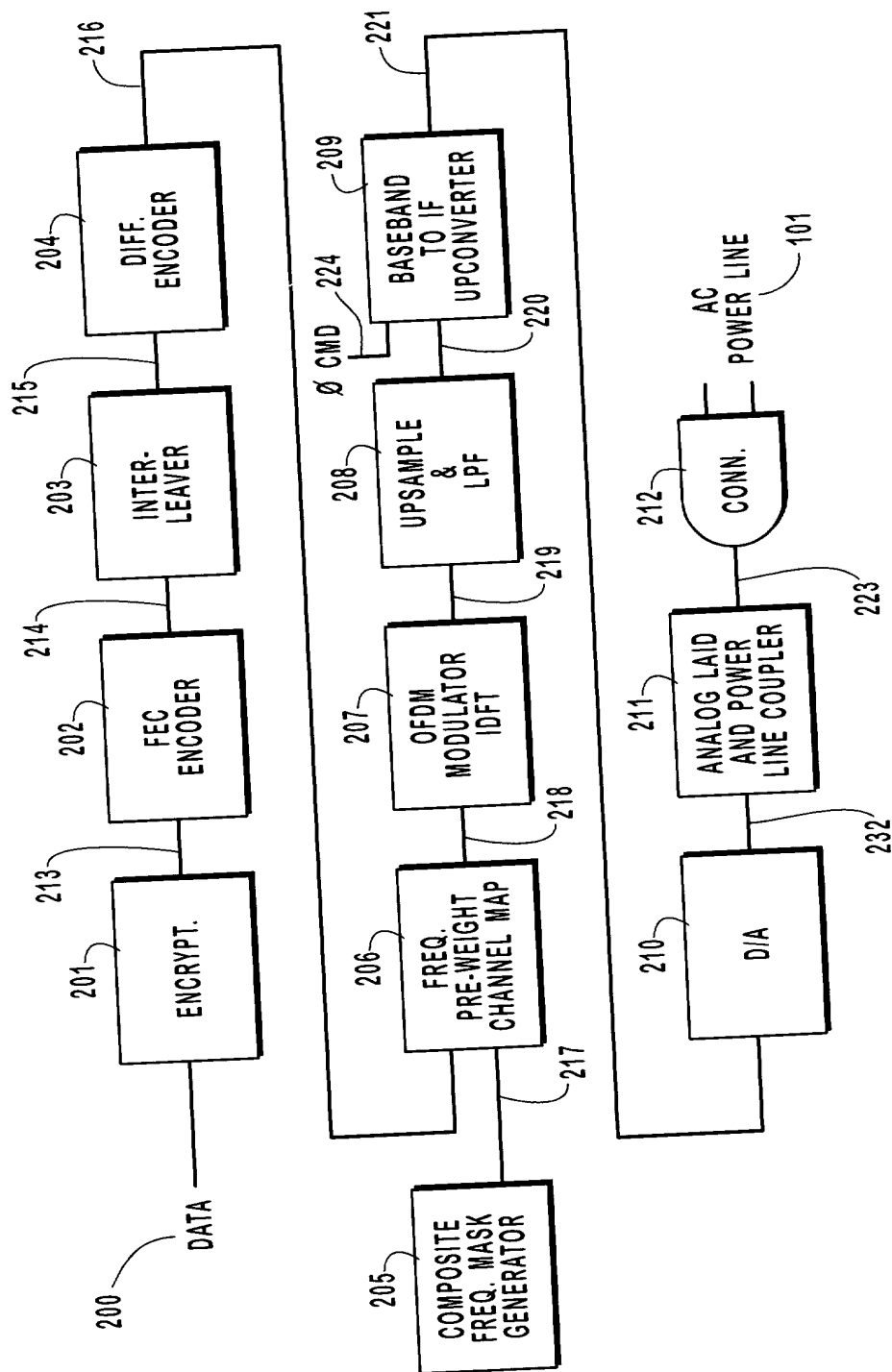


FIG. 1



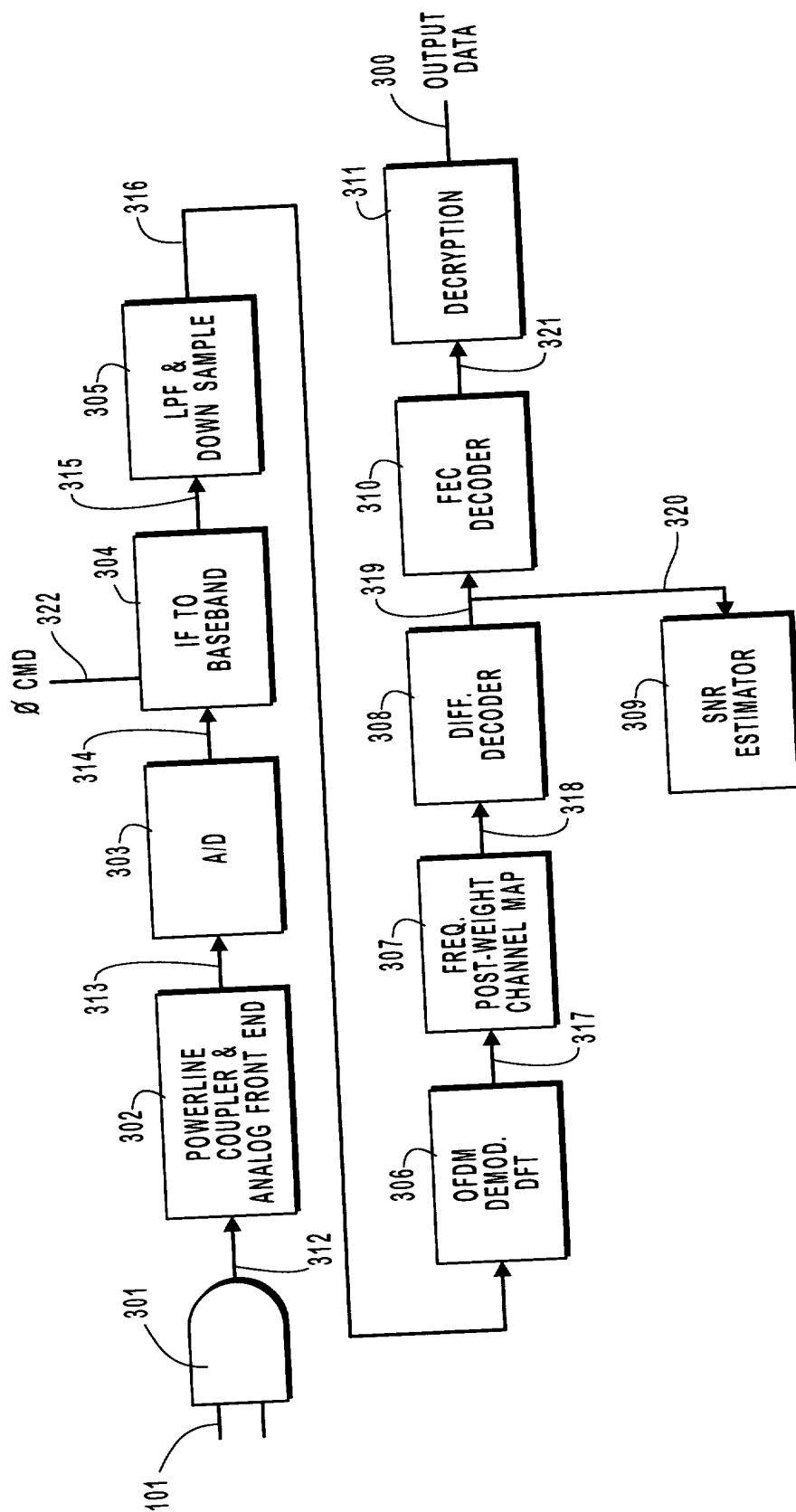


FIG. 3

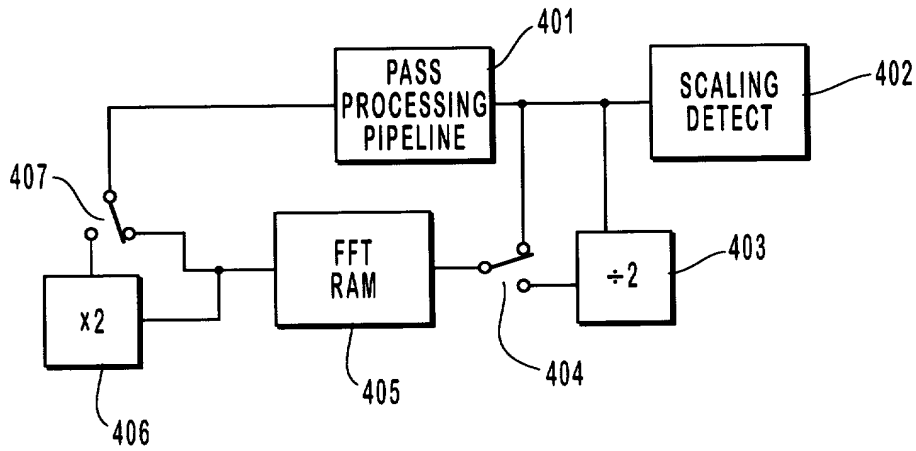


FIG. 4

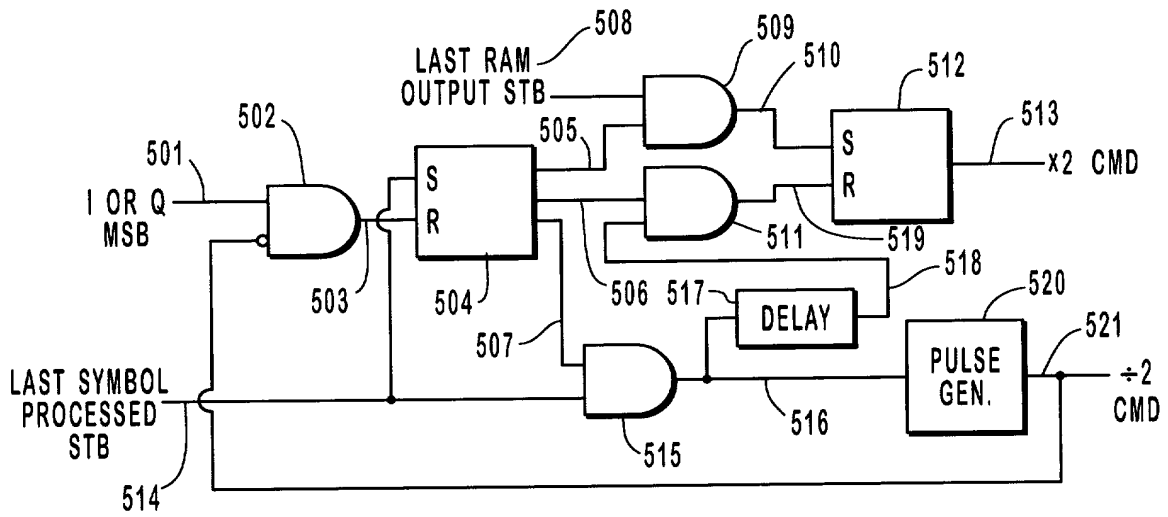


FIG. 5

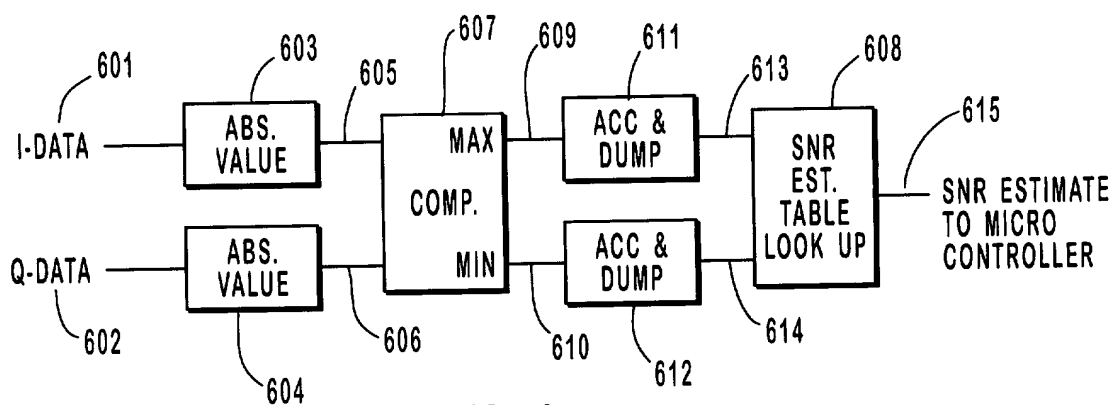


FIG. 6

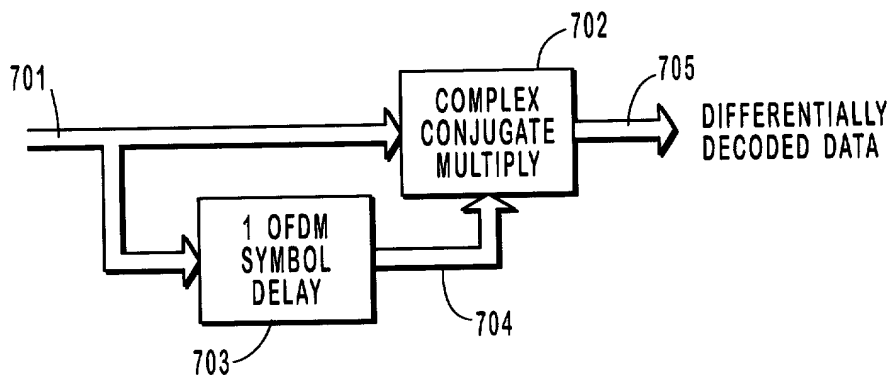


FIG. 7

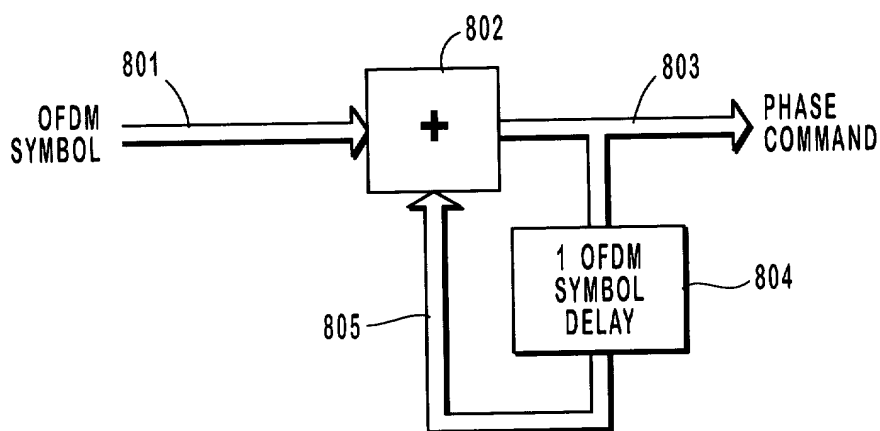


FIG. 8

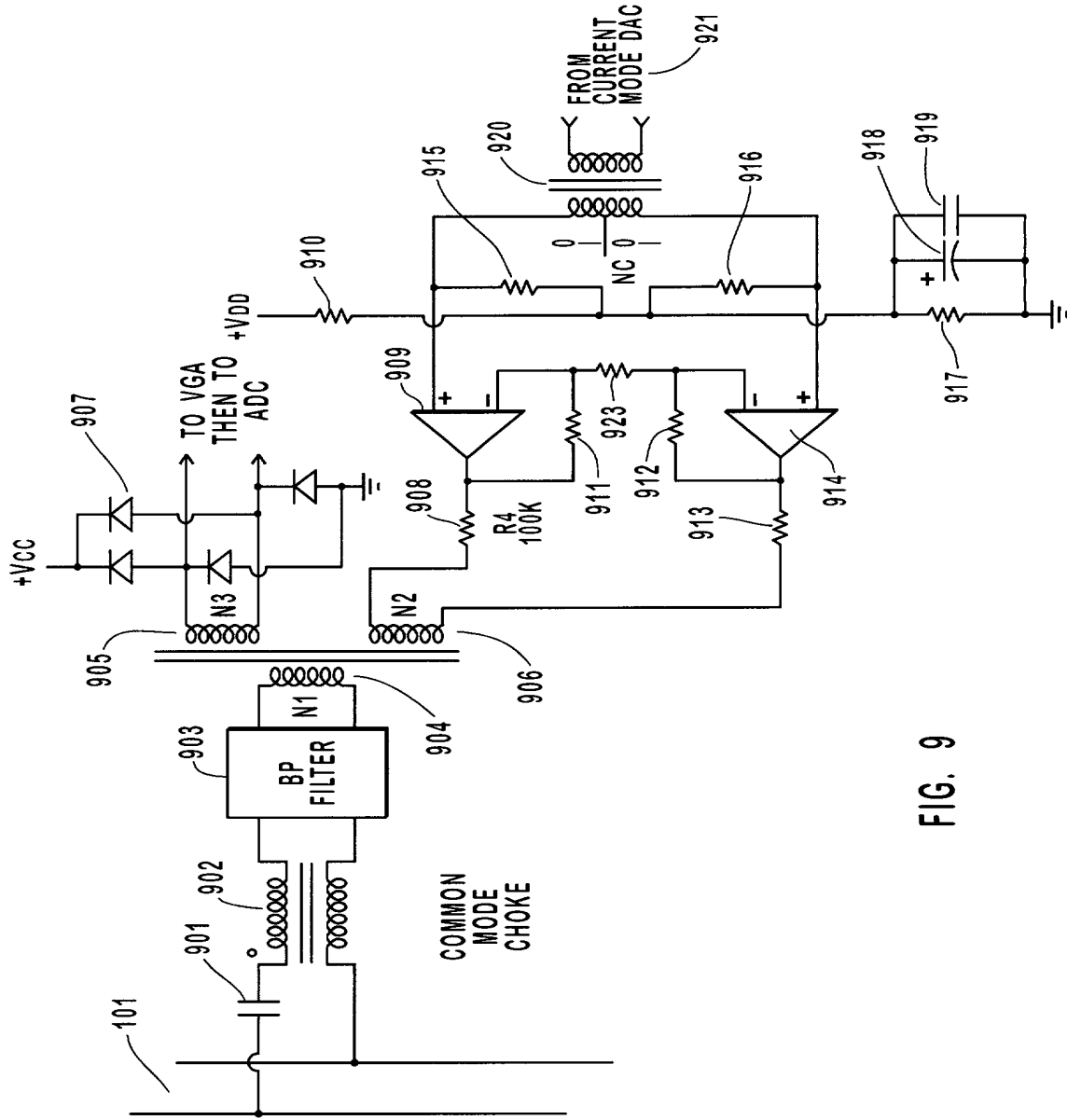


FIG. 9

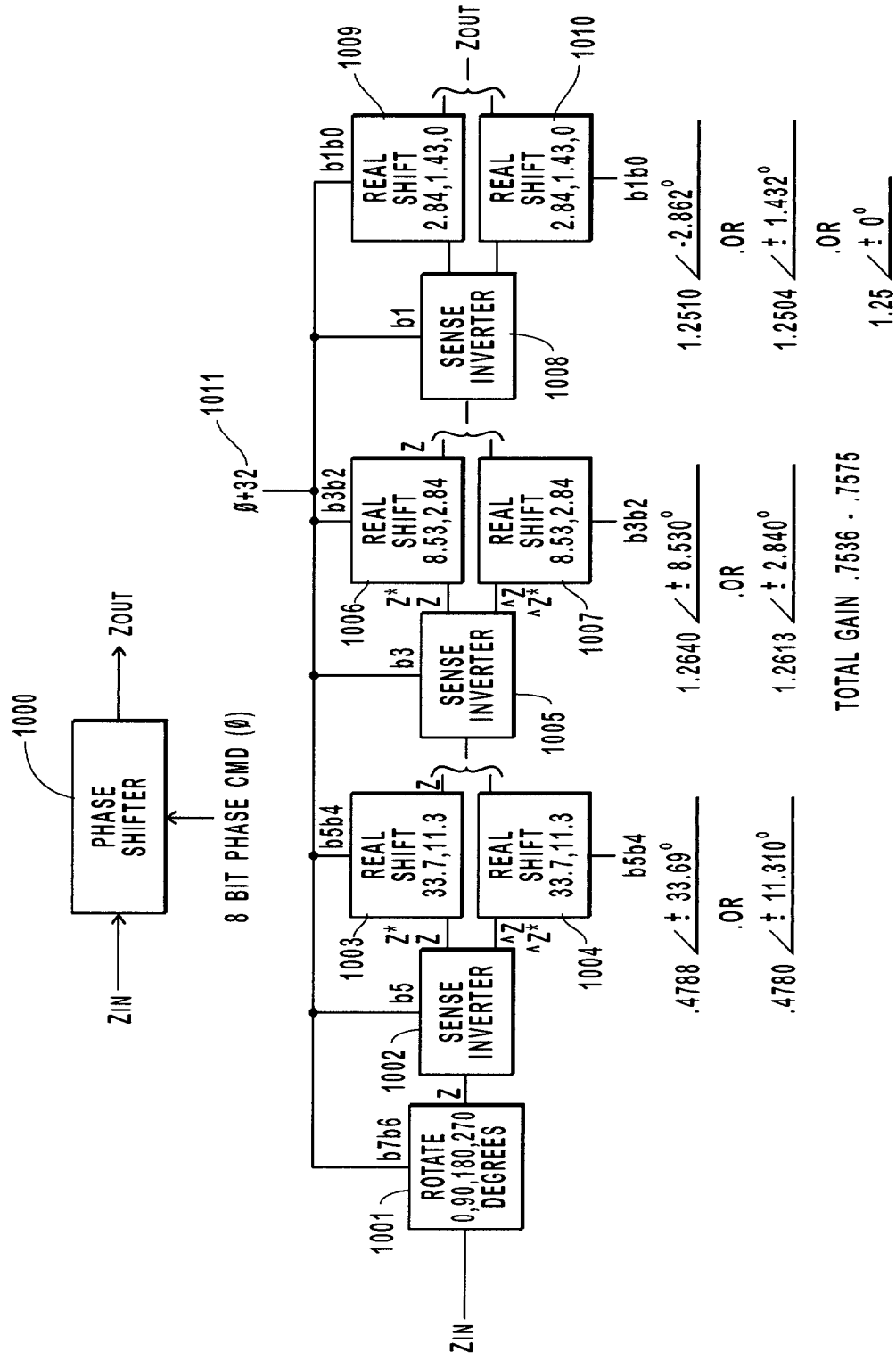


FIG. 10

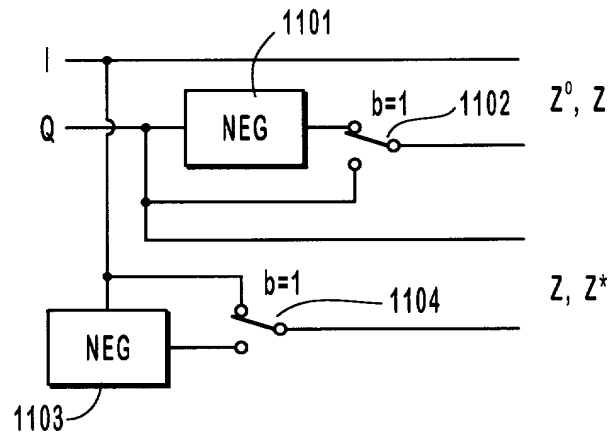


FIG. 11

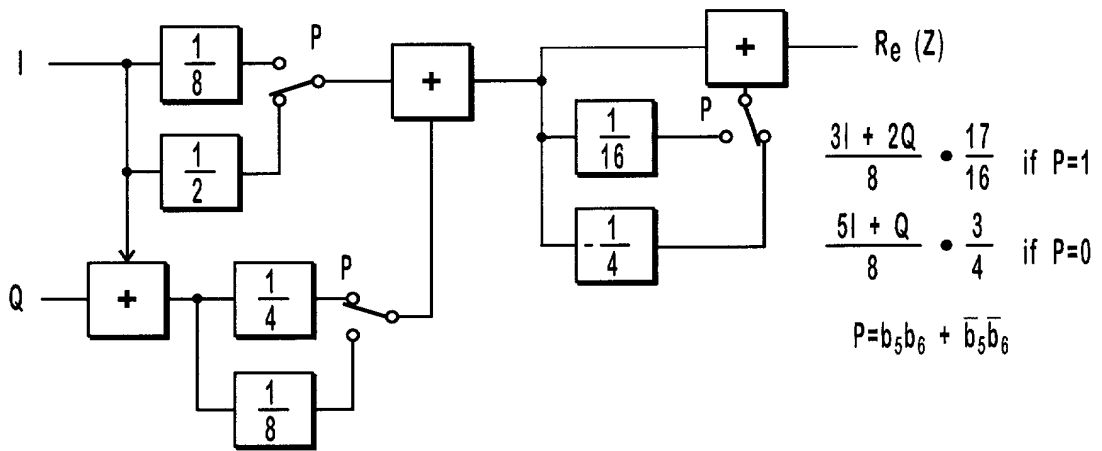


FIG. 12

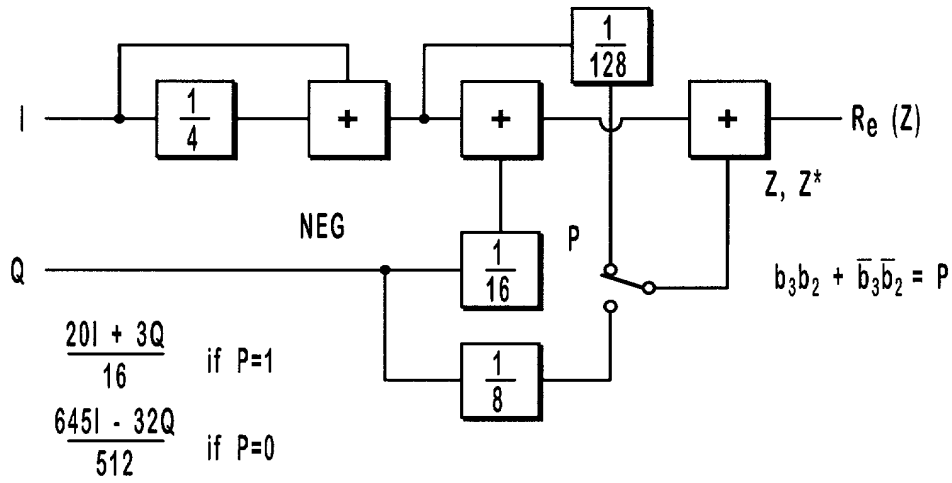


FIG. 13

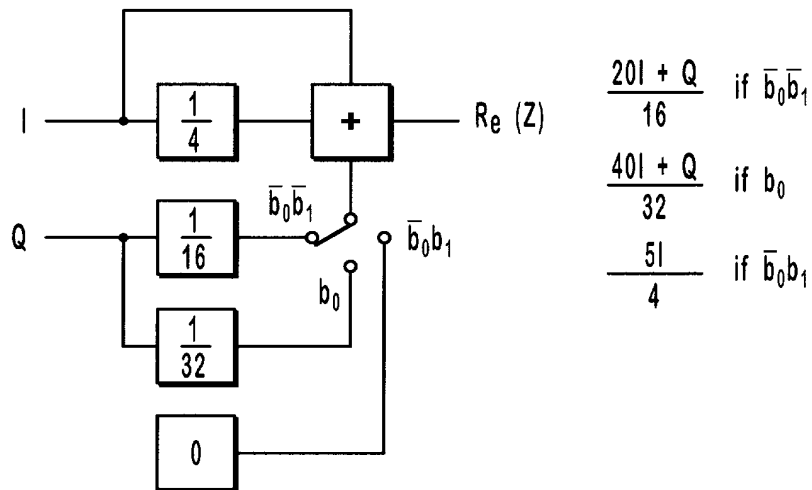


FIG. 14

```

function z= pshift1(z,th);
iz=real(z); qz=imag(z);
tha=2*(bitand(th+32,2.^(0:7))>0)-1; %make array of phase bits + 32

%0,90,180,270
if tha(7)= =1;z=j*z;end;
if tha(8)= =1;z= -z; end;

% + - 33.69,11.31
if tha(6)*tha(5)= = -1;
    z=3/16*(5+tha(6)*j)*z;
else;
    z=17/64*(3+tha(6)*2*j)*z;
end;

% +- 8.53,2.84
if tha(4)*tha(3)= =-1;
    z=(20*(1+1/128) + tha(4)*j)*z/32;
else;
    z=(20+tha(4)*3*j)*z/32;
end;

% -2.86 -1.43 0 1.43
if tha(1)= =1;
    z=(40+tha(2)*j)*z/32;
elseif tha(2)= = -1;
    z=(20-j)*z/16;
else;
    z=5*z/4;
end;

```

FIG. 15

106290" 47956860

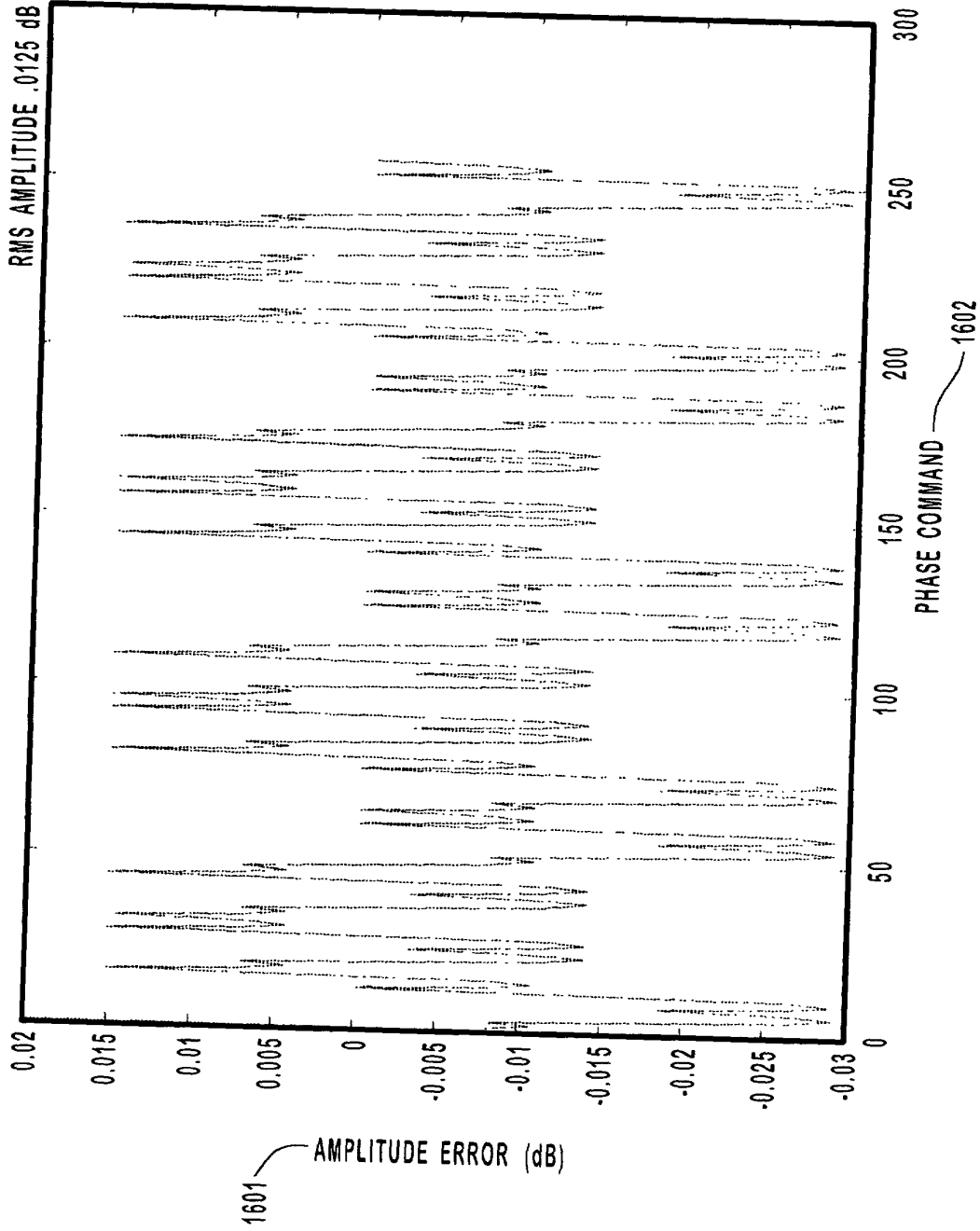


FIG. 16

FIG. 17

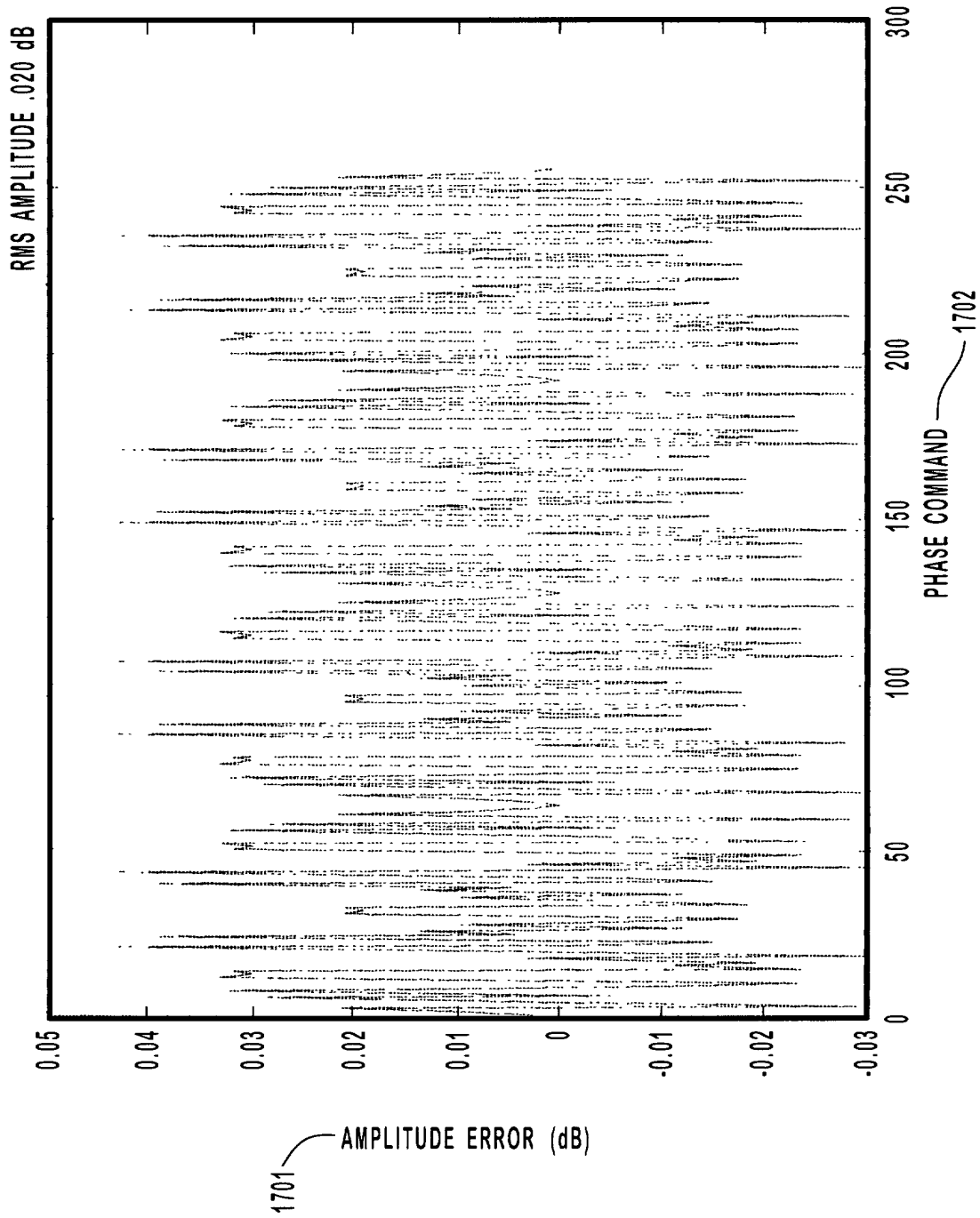


FIG. 17

106290" 4756660

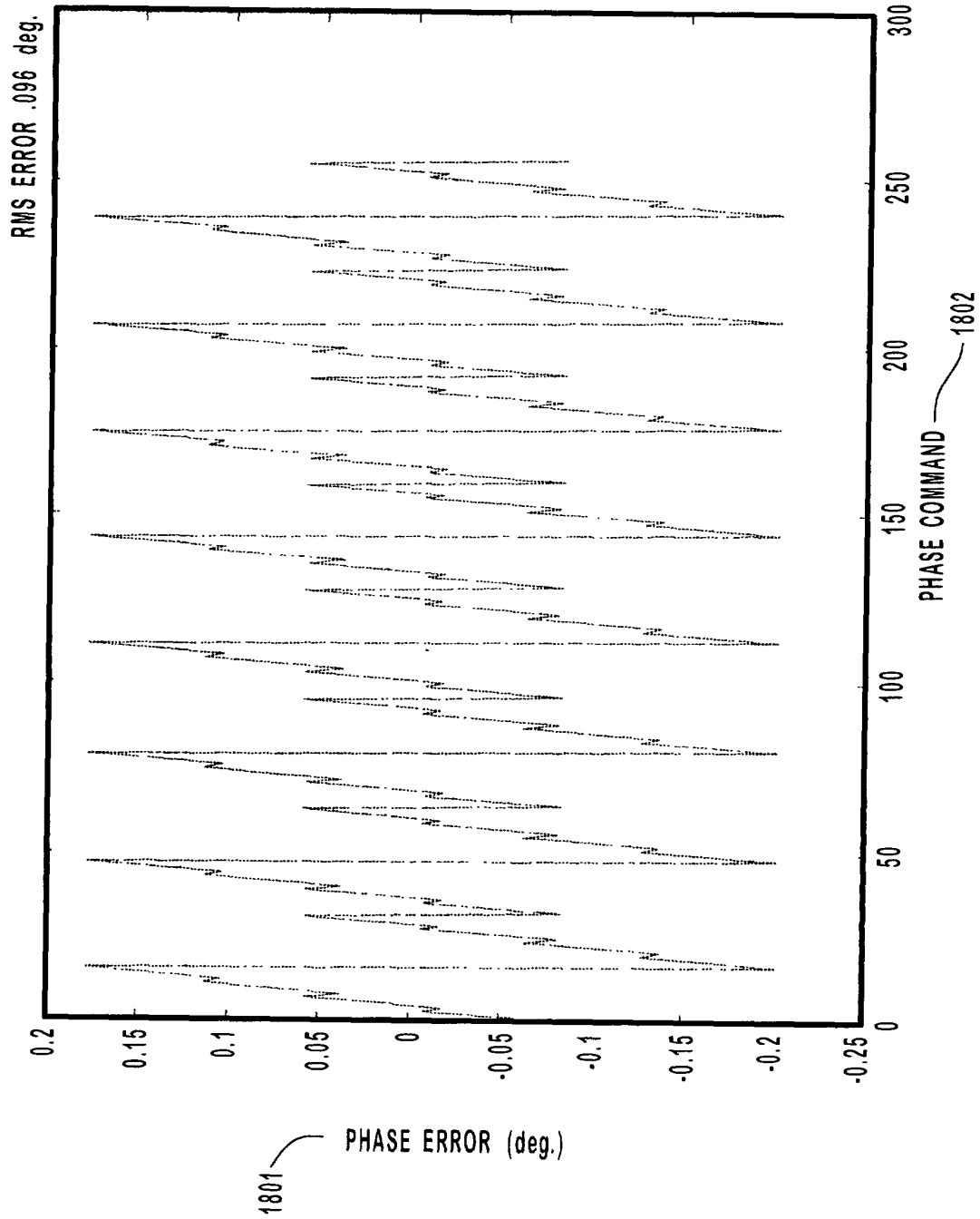


FIG. 18

06290"4956860

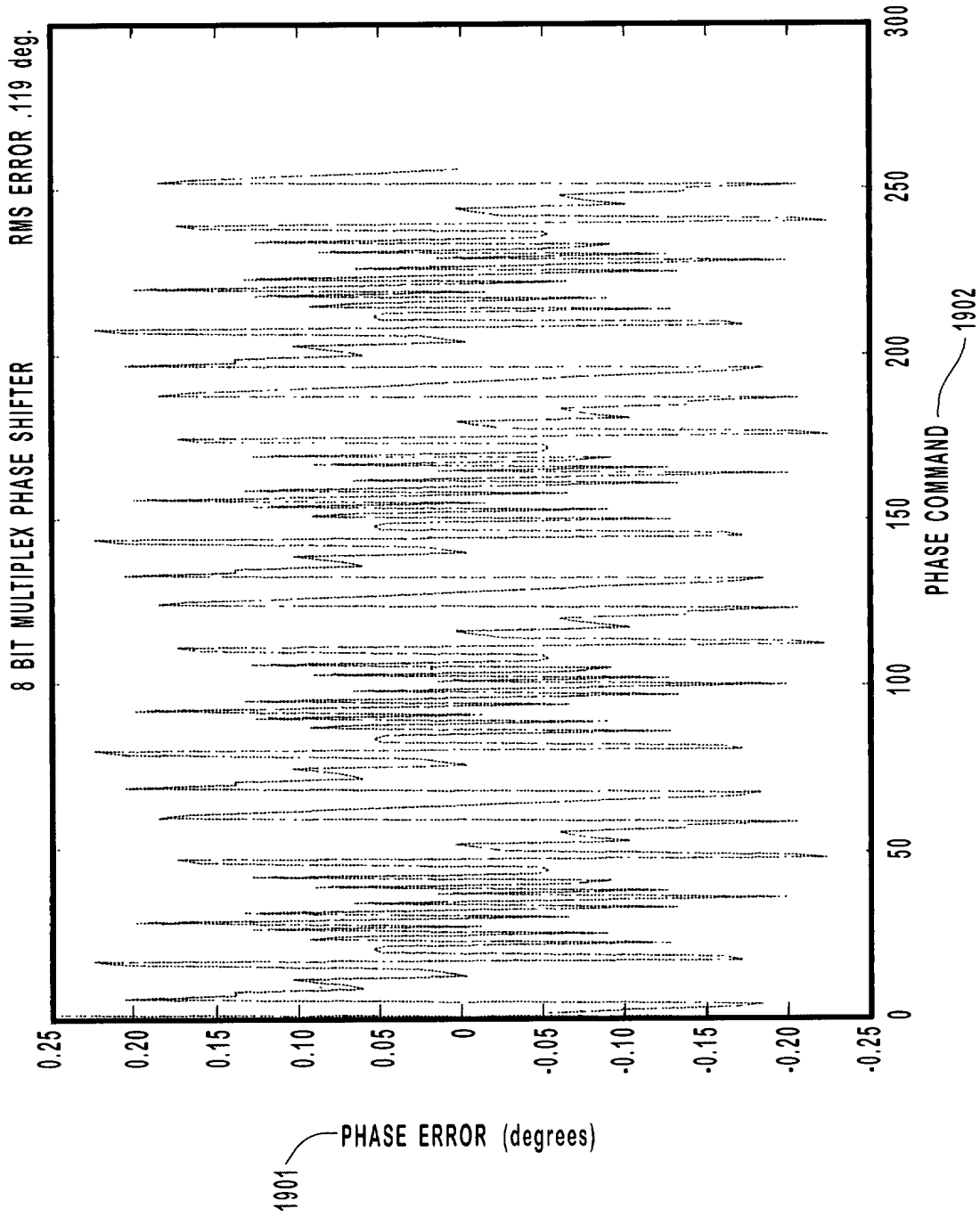


FIG. 19

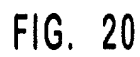
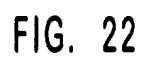
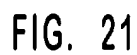


FIG. 20



17 / 68 Anatomy of the FFT

This is a 256 point base 2 example of an FFT (a fast dft)

$k := 0.. 255$ FFT index

$x_k := \text{rnd}(1) + j \cdot \text{rnd}(1)$ FFT dummy argument

The discrete forrier transform takes 256^2 operations

$$\text{DFT}_k := \sum_{i=0}^{255} x_i \cdot e^{j \cdot 2\pi \cdot \frac{i \cdot k}{256}}$$

If we combine terms from the first and 2nd half of the summation we have

$$\sum_{i=0}^{127} \left(x_i + x_{i+128} \cdot e^{j \cdot 2\pi \cdot \frac{k}{2}} \right) \cdot e^{j \cdot 2\pi \cdot \frac{i \cdot k}{256}}$$

Note the term in paremphases runs over half of N and is only unique for $k \bmod 2$
so:

$i1 := 0.. 127$ $k1 := 0.. 1$

$$X1_{i1, k1} := x_{i1} + x_{i1+128} \cdot e^{(j \cdot 2\pi) \cdot \frac{k1}{2}}$$

Then

$$\text{DFT1}_k := \sum_{i=0}^{127} X1_{i, \text{mod}(k, 2)} \cdot e^{j \cdot 2\pi \cdot \frac{i \cdot k}{256}} \quad \sum_k \left(\left| \text{DFT}_k - \text{DFT1}_k \right| \right)^2 = 0$$

This operation only takes half the DFT steps by taking 128 steps to precompute X1

We can further reduce the computational load by doing the same thing again

$i1 := 0.. 63$ $k1 := 0.. 3$

$$X2_{i1, k1} := X1_{i1, \text{mod}(k1, 2)} + X1_{i1+64, \text{mod}(k1, 2)} \cdot e^{(j \cdot 2\pi) \cdot \frac{k1}{4}}$$

Then

$$\text{DFT2}_k := \sum_{i=0}^{63} X2_{i, \text{mod}(k, 4)} \cdot e^{j \cdot 2\pi \cdot \frac{i \cdot k}{256}} \quad \sum_k \left(\left| \text{DFT}_k - \text{DFT2}_k \right| \right)^2 = 0$$

FIG. 23a

We repeat this operation a total of 8 times until i ranges only over zero
We show one more trick. Instead of:

$$i1 := 0..31 \quad k1 := 0..7$$

$$X3_{i1,k1} := X2_{i1, \text{mod}(k1,4)} + X2_{i1+32, \text{mod}(k1,4)} \cdot e^{(j \cdot 2\pi) \frac{k1}{8}}$$

Use

$$k1 := 0..3 \quad t_{i1,k1} := X2_{i1+32,k1} \cdot e^{(j \cdot 2\pi) \frac{k1}{8}}$$

$$X3_{i1,k1} := X2_{i1,k1} + t_{i1,k1} \quad X3_{i1,k1+4} := X2_{i1,k1} - t_{i1,k1}$$

It only requires half the phase shifts. This operation is called a **butterfly**
Continuing

$$i1 := 0..15 \quad k1 := 0..7 \quad t_{i1,k1} := X3_{i1+16,k1} \cdot e^{(j \cdot 2\pi) \frac{k1}{16}}$$

$$X4_{i1,k1} := X3_{i1,k1} + t_{i1,k1} \quad X4_{i1,k1+8} := X3_{i1,k1} - t_{i1,k1}$$

$$i1 := 0..7 \quad k1 := 0..15 \quad t_{i1,k1} := X4_{i1+8,k1} \cdot e^{(j \cdot 2\pi) \frac{k1}{32}}$$

$$X5_{i1,k1} := X4_{i1,k1} + t_{i1,k1} \quad X5_{i1,k1+16} := X4_{i1,k1} - t_{i1,k1}$$

$$i1 := 0..3 \quad k1 := 0..31 \quad t_{i1,k1} := X5_{i1+4,k1} \cdot e^{(j \cdot 2\pi) \frac{k1}{64}}$$

$$X6_{i1,k1} := X5_{i1,k1} + t_{i1,k1} \quad X6_{i1,k1+32} := X5_{i1,k1} - t_{i1,k1}$$

$$i1 := 0..1 \quad k1 := 0..63 \quad t_{i1,k1} := X6_{i1+2,k1} \cdot e^{(j \cdot 2\pi) \frac{k1}{128}}$$

$$X7_{i1,k1} := X6_{i1,k1} + t_{i1,k1} \quad X7_{i1,k1+64} := X6_{i1,k1} - t_{i1,k1}$$

and finally

$$k1 := 0..127 \quad t_{0,k1} := X7_{1,k1} \cdot e^{(j \cdot 2\pi) \frac{k1}{256}}$$

$$X8_{k1} := X7_{0,k1} + t_{0,k1} \quad X8_{k1+128} := X7_{0,k1} - t_{0,k1}$$

$$\sum_k \left(\left\| \text{DFT}_k - X8_k \right\| \right)^2 = 0$$

FIG. 23b

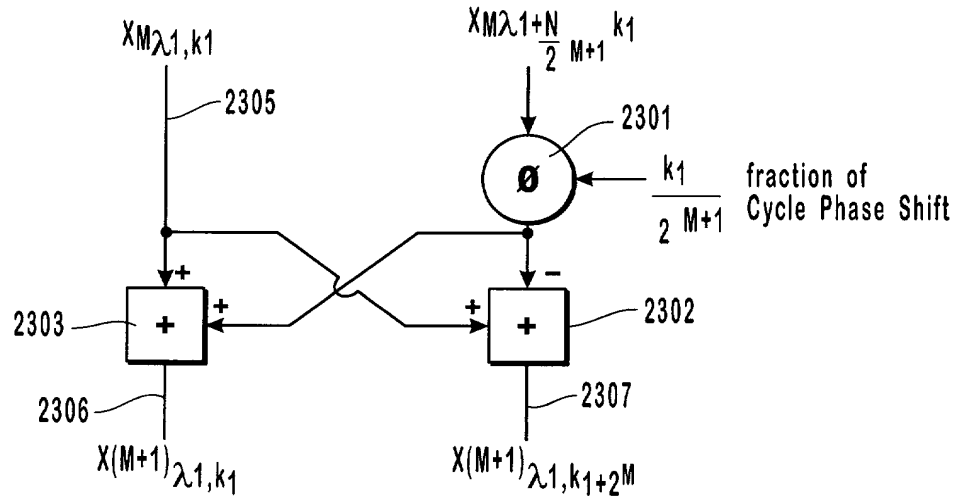


FIG. 23c

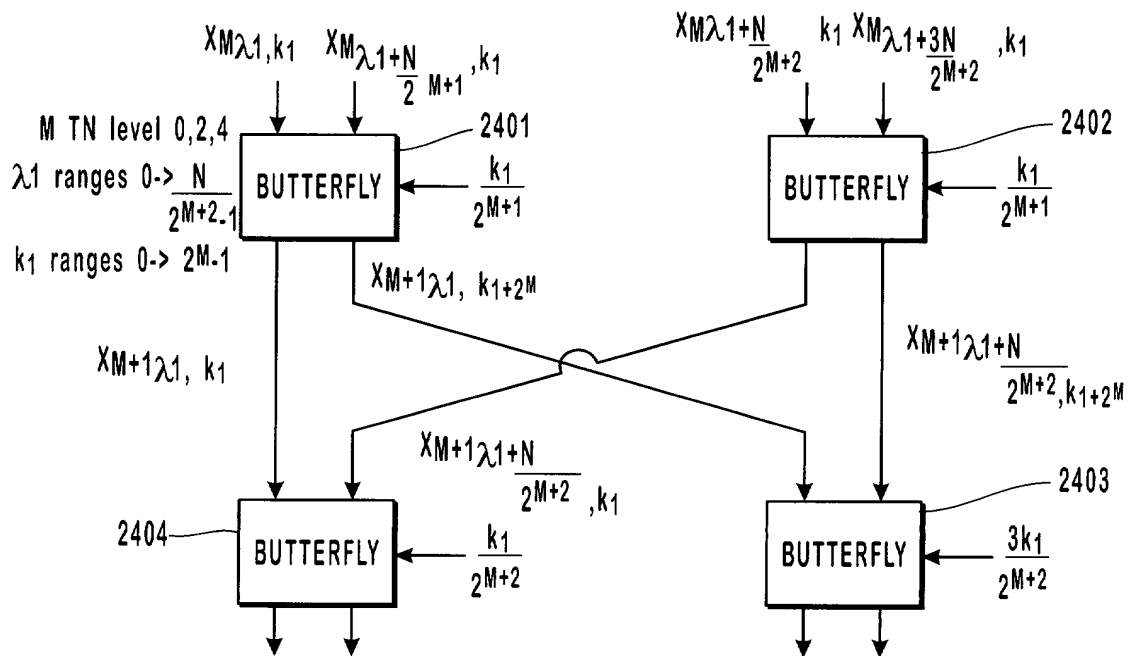


FIG. 24

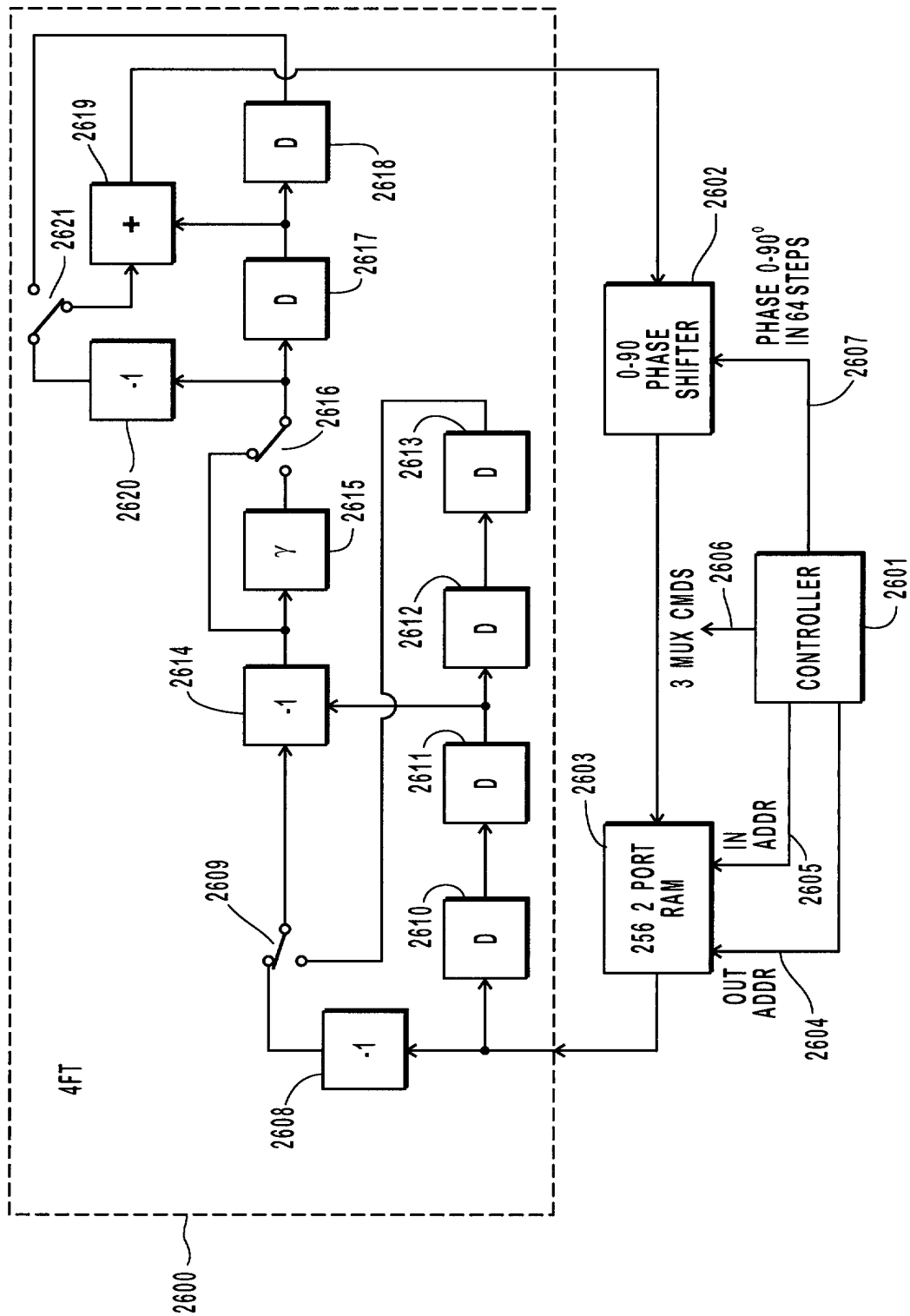


FIG. 26

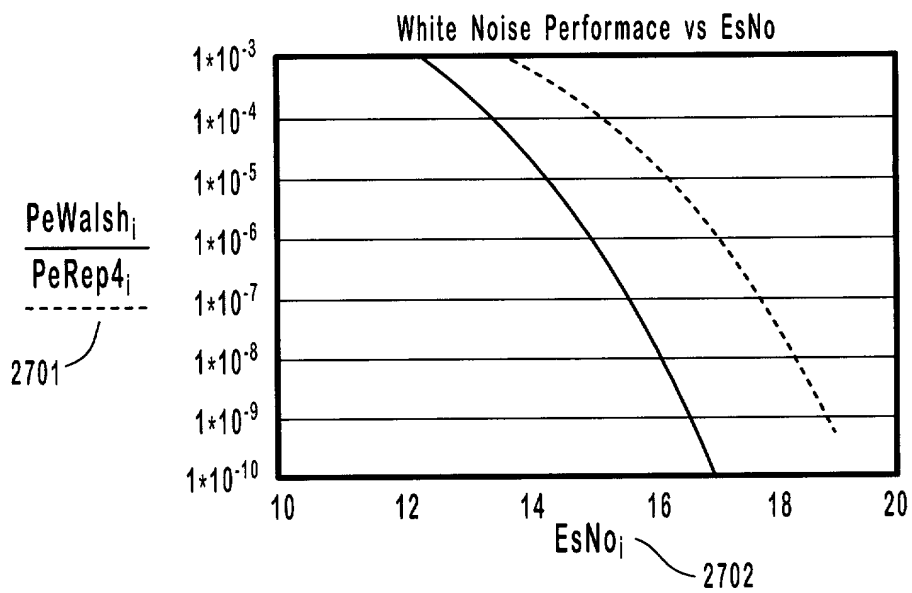


FIG. 27a

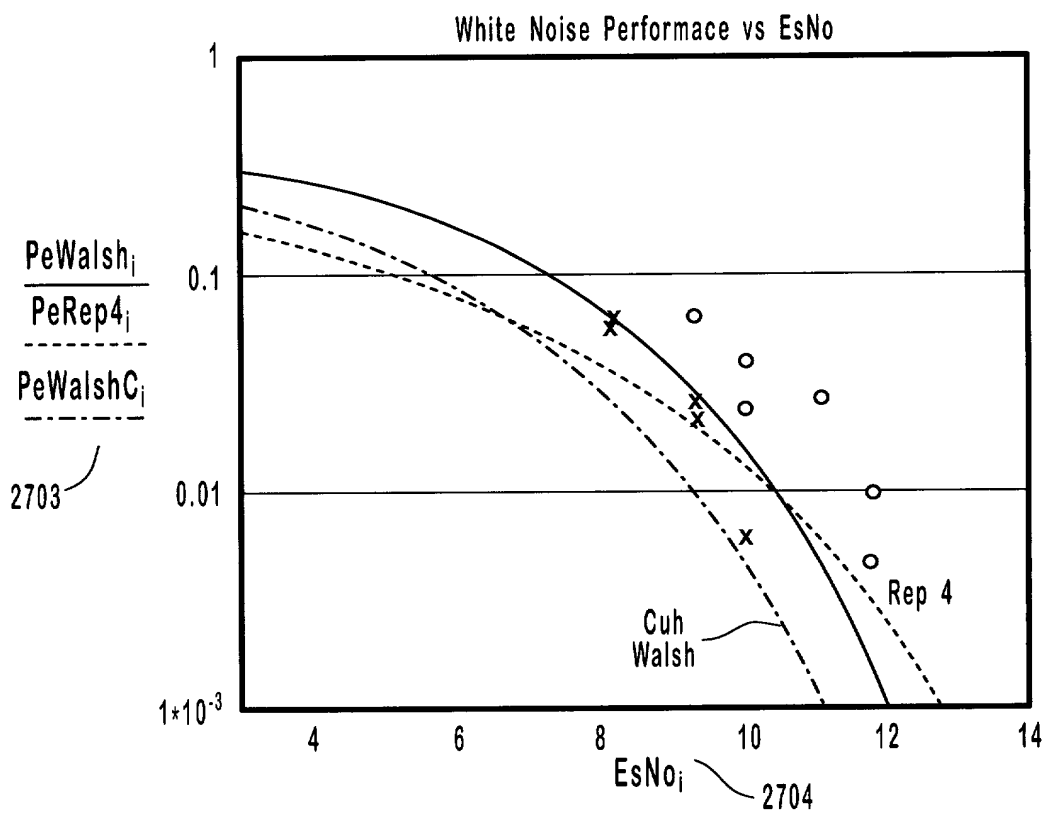


FIG. 27b

```

% finds likelihood of 16ary symbol error for nbe erasures
tr={ 1 -1; 1 1}; tr=[tr -tr; tr tr]; tr=[tr -tr; tr tr]; tr=[tr -tr; tr tr];
v=zeros(1,16);
nsym=zeros(1,16);
while sum(v)<16;
    [a,n]=max (tr*v');
    if n<16;
        nbe=sum(v==0);
        nsym(nbe)=nsym(nbe) +1;
    end;
    k=1;
    v(k)= -(v(k)-1); % -1 or 0 – erasures
    while v(k)==0;
        k=k+1;
        v(k)= -(v(k)-1); % -1 or 0 – erasures
    end;
end;
pe=.005:.005:.5;
for k=1:100;
    we(k)=sum(pe(k).^(1:16).*(1-pe(k)).^(15:-1:0).*nsym);
end;
rhe=1-((1-pe).^4+4*pe.*(1-pe).^3).^4; % repeat sym hard error
rea=1-(1-pe.^4).64; % repeat sym erasure error
plot (pe,we,pe,rea)

```

FIG. 28

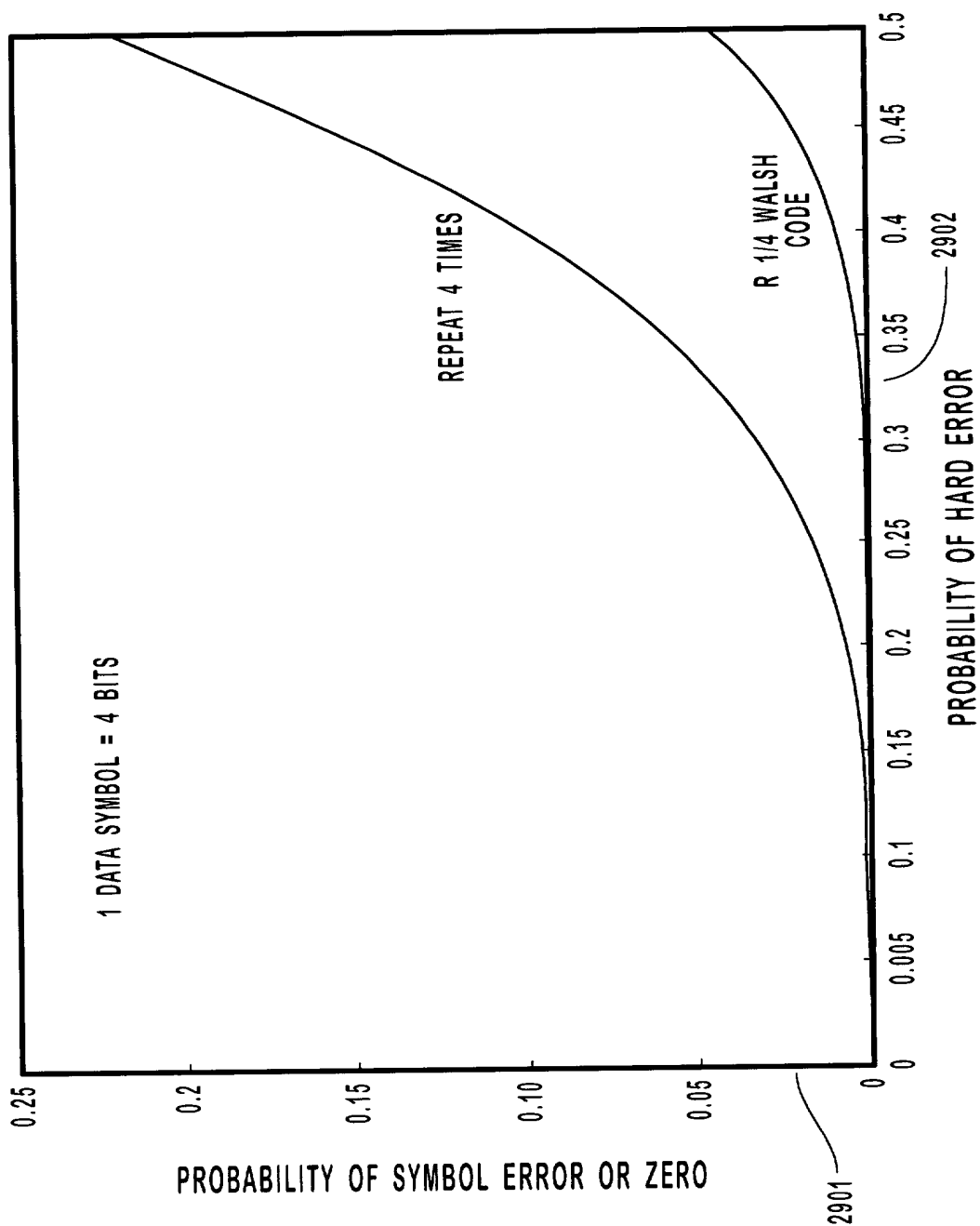


FIG. 29

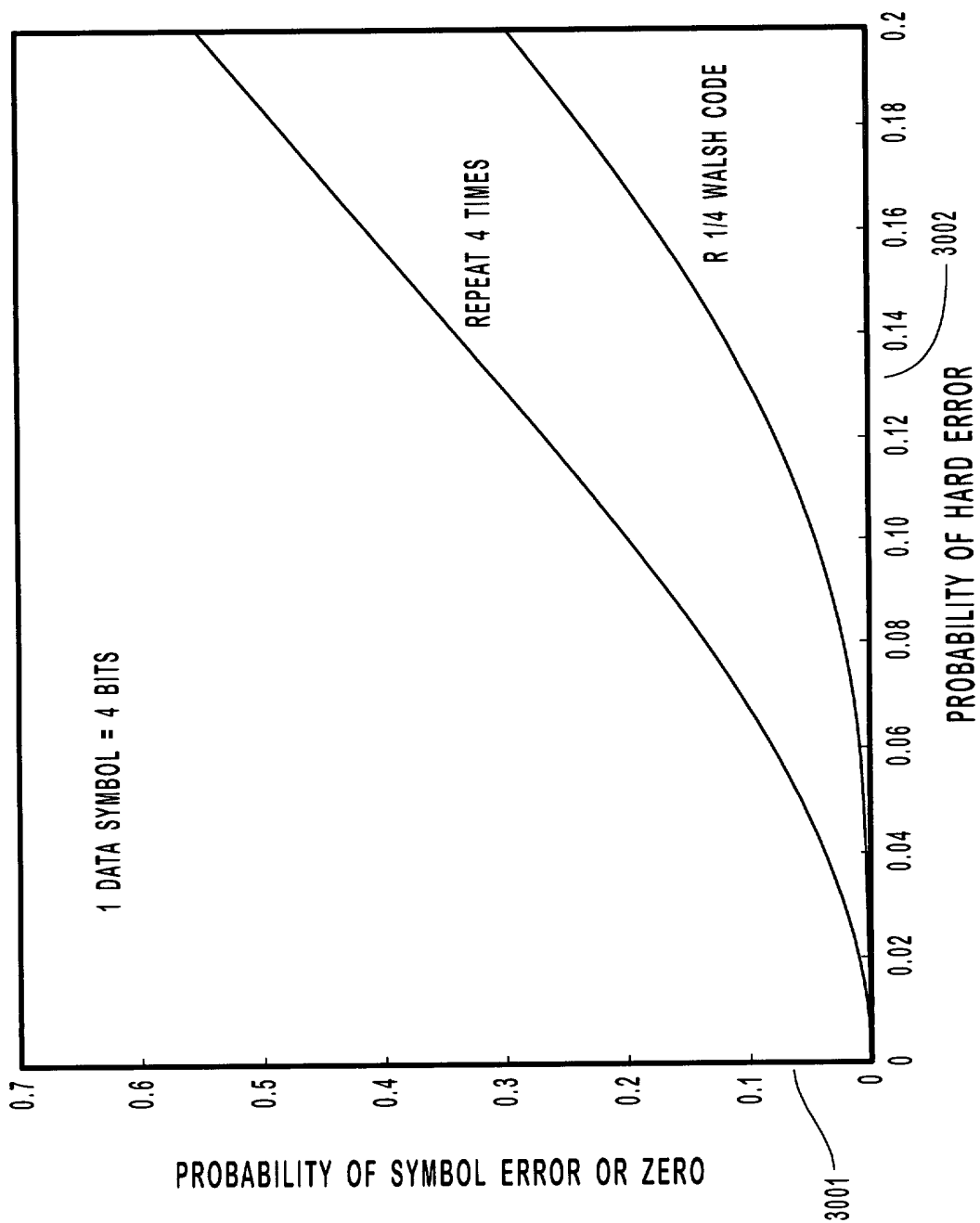


FIG. 30

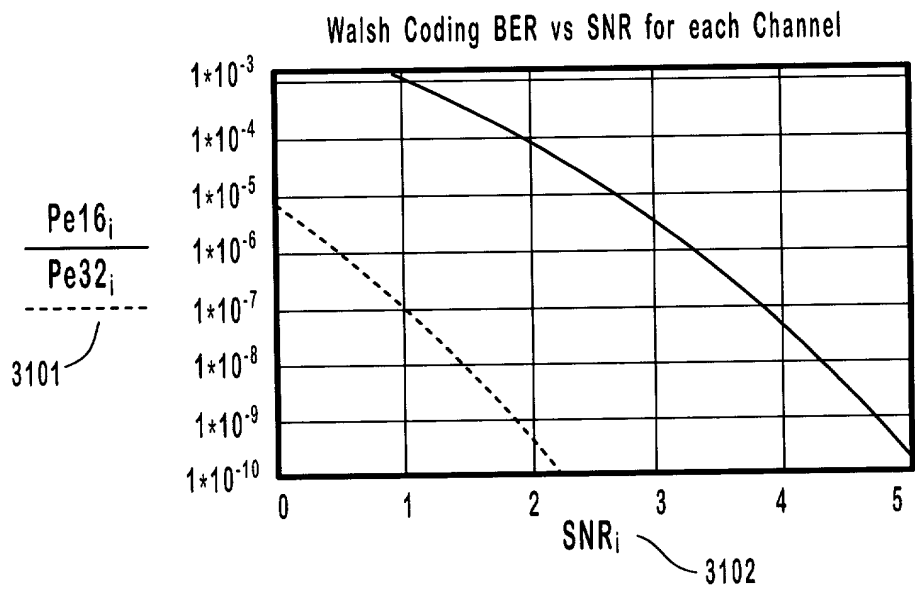


FIG. 31

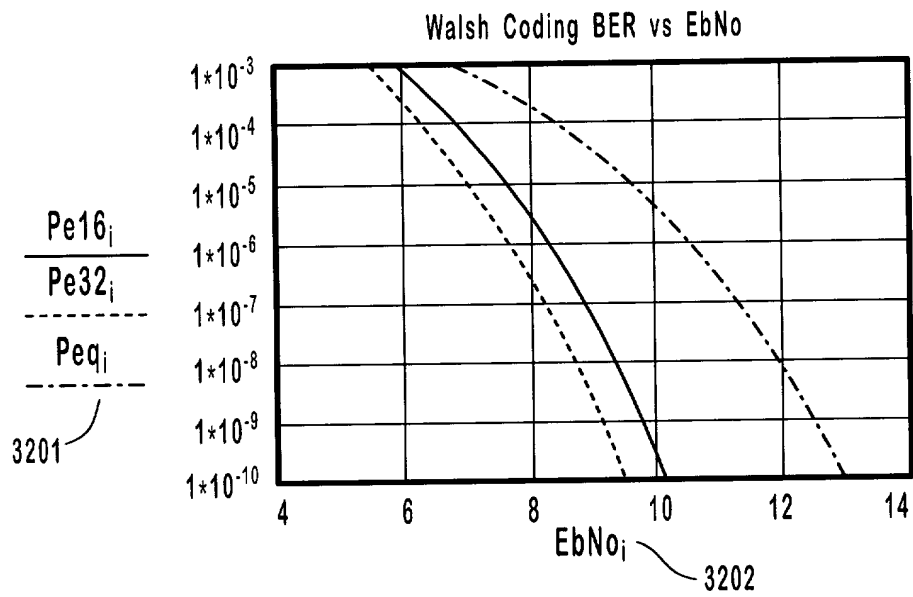


FIG. 32

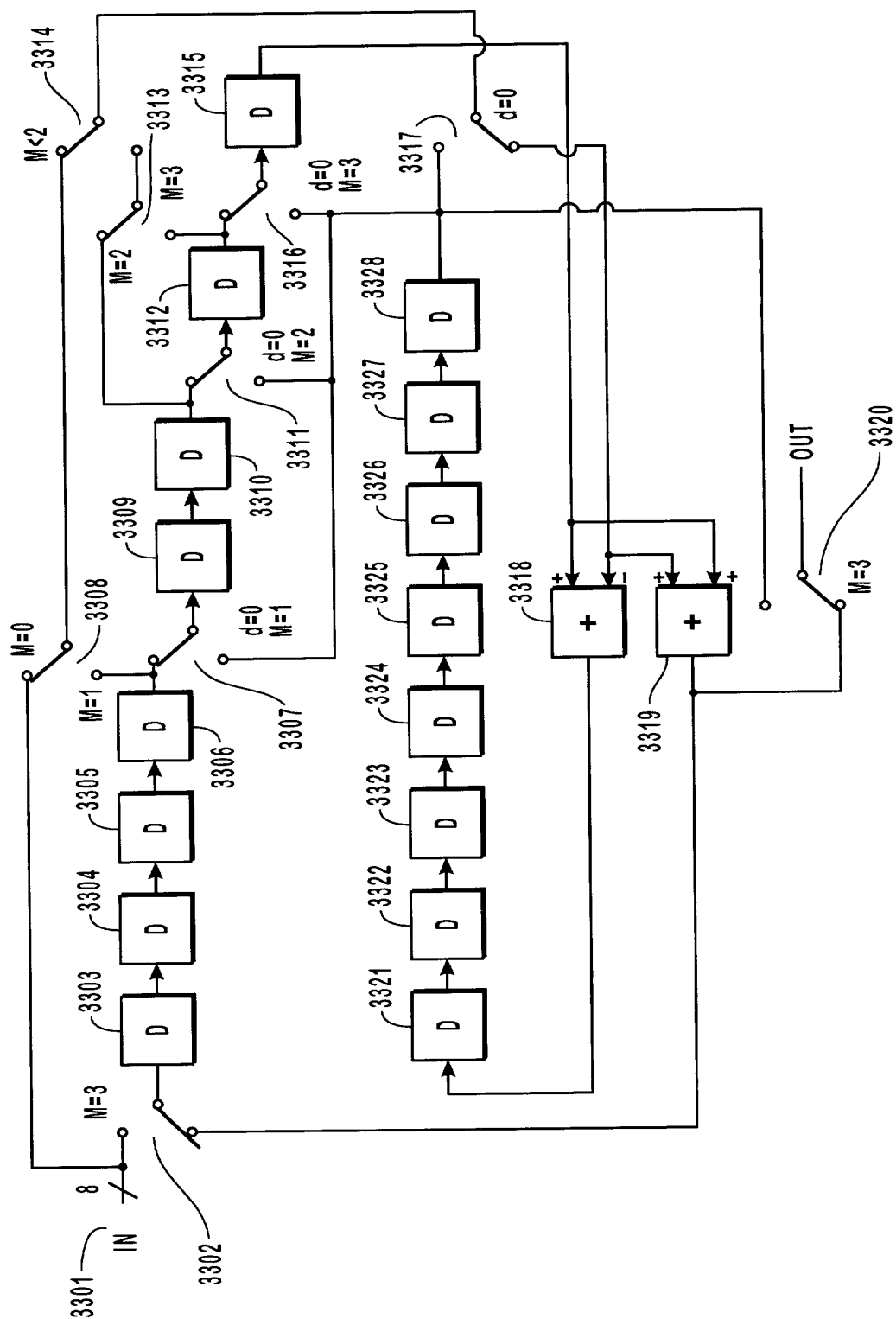


FIG. 33

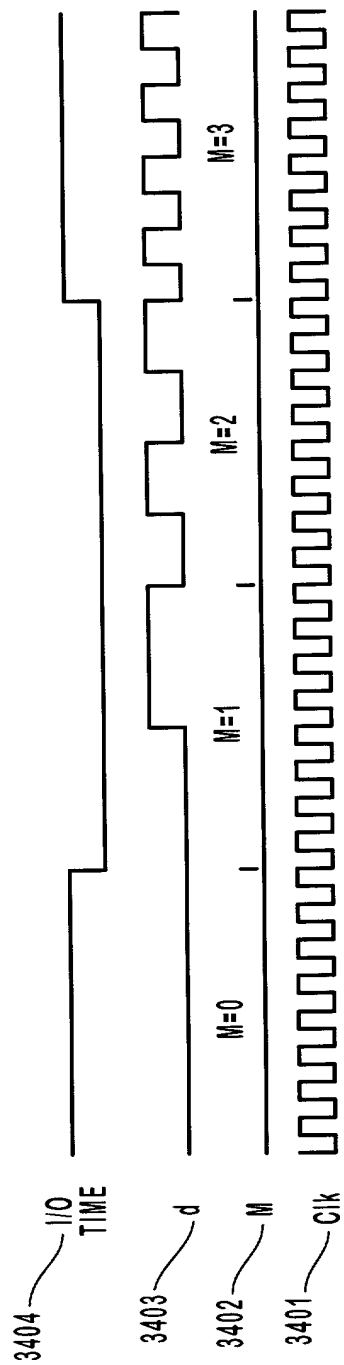


FIG. 34

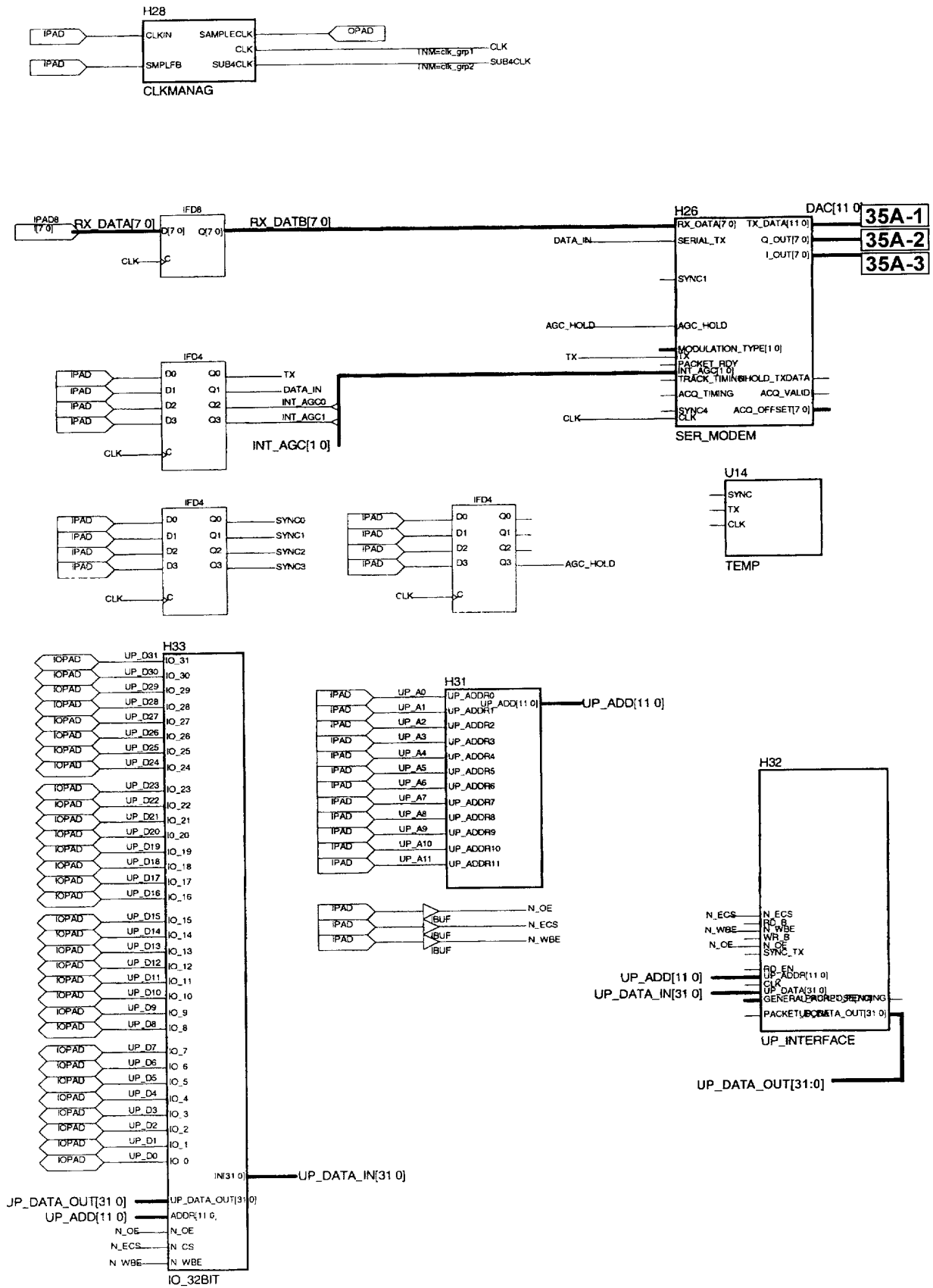


FIG. 35a-1

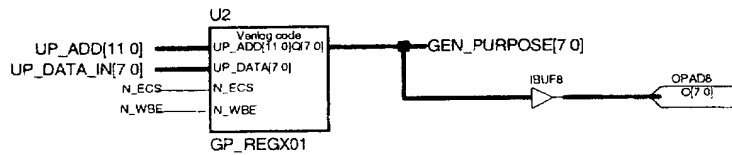
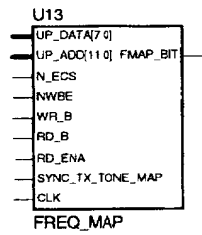
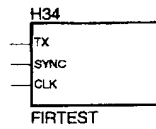
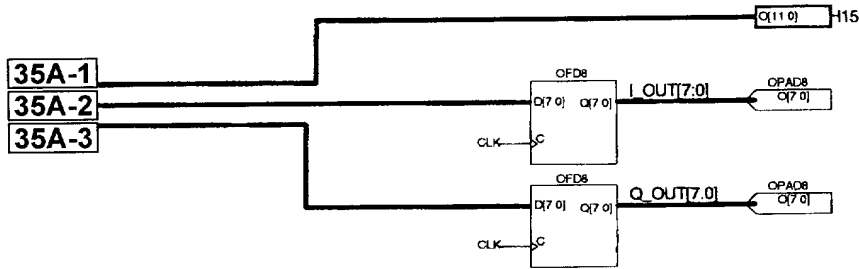


FIG. 35a-2

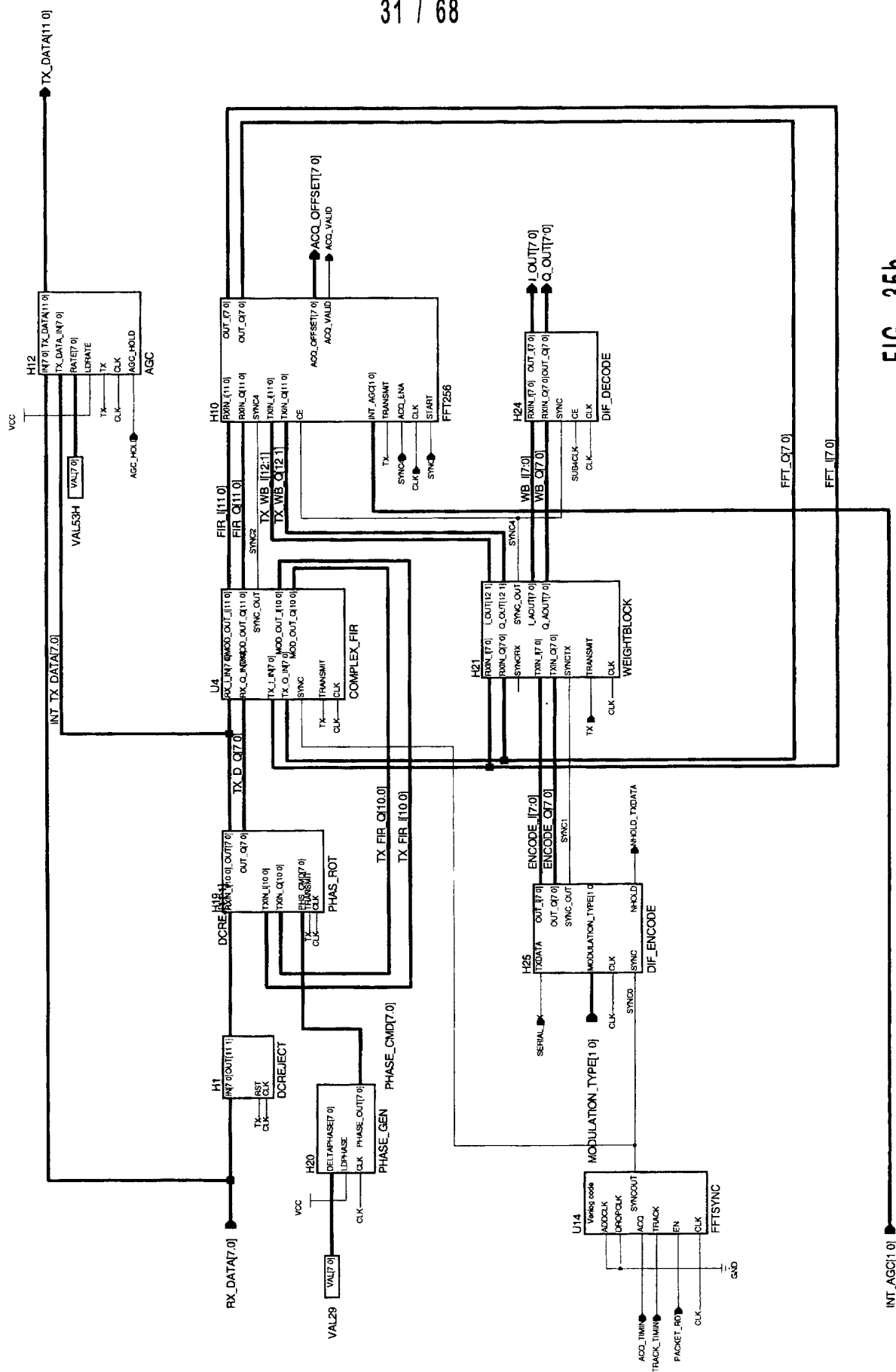


FIG. 35b

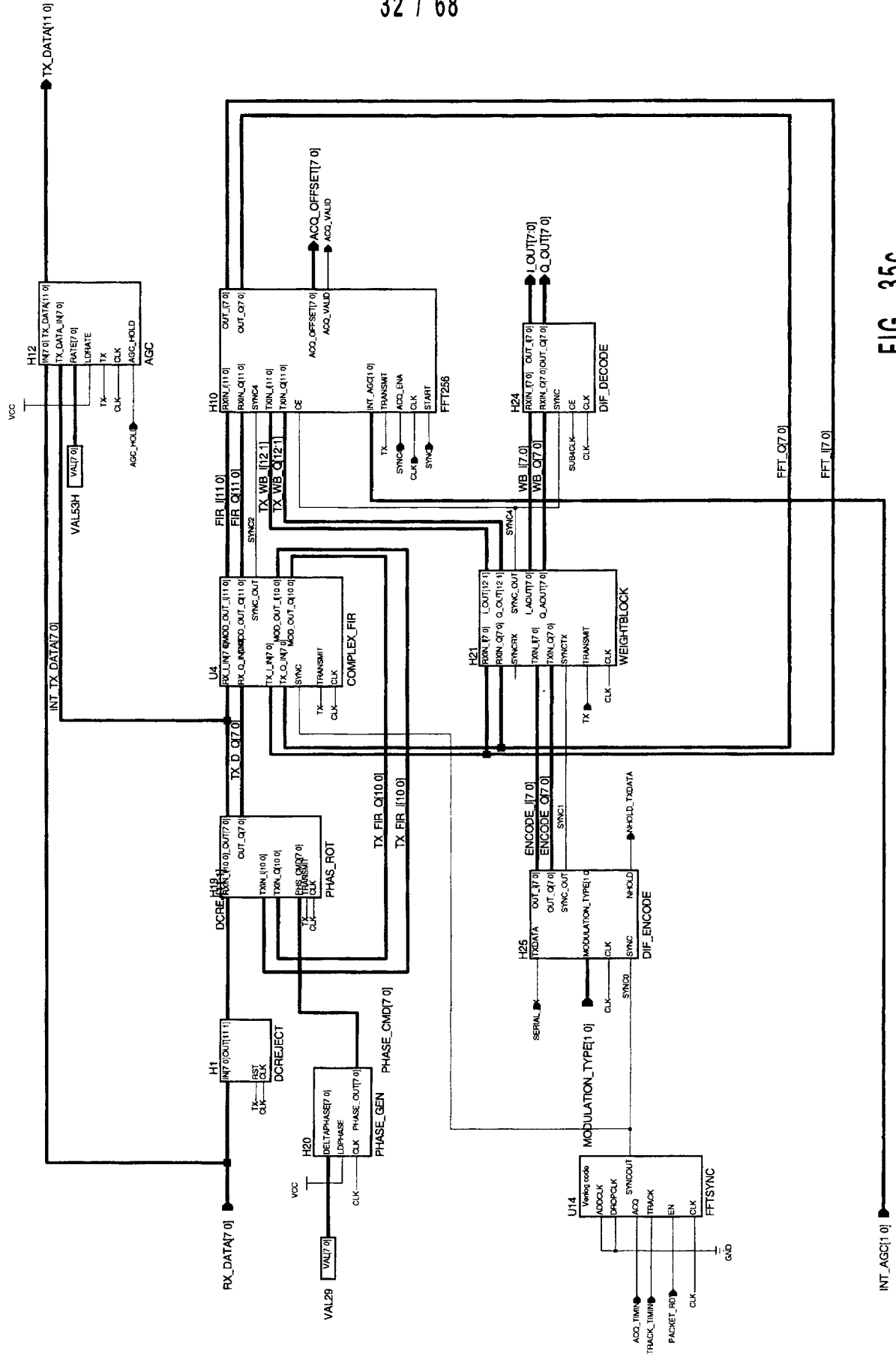


FIG. 35C

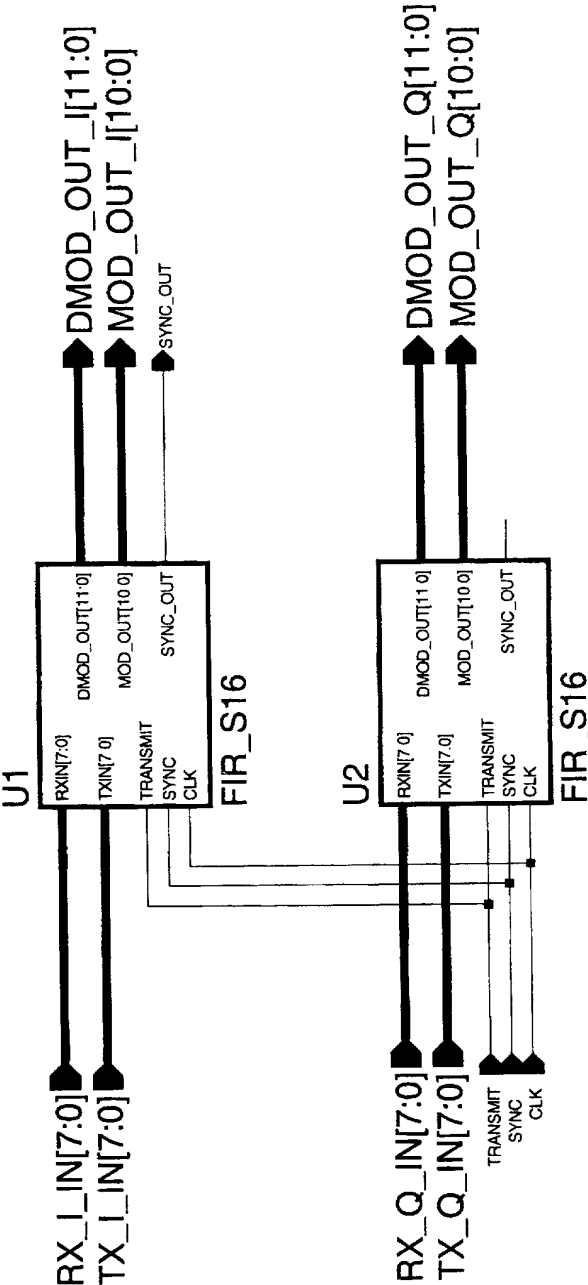


FIG. 35d

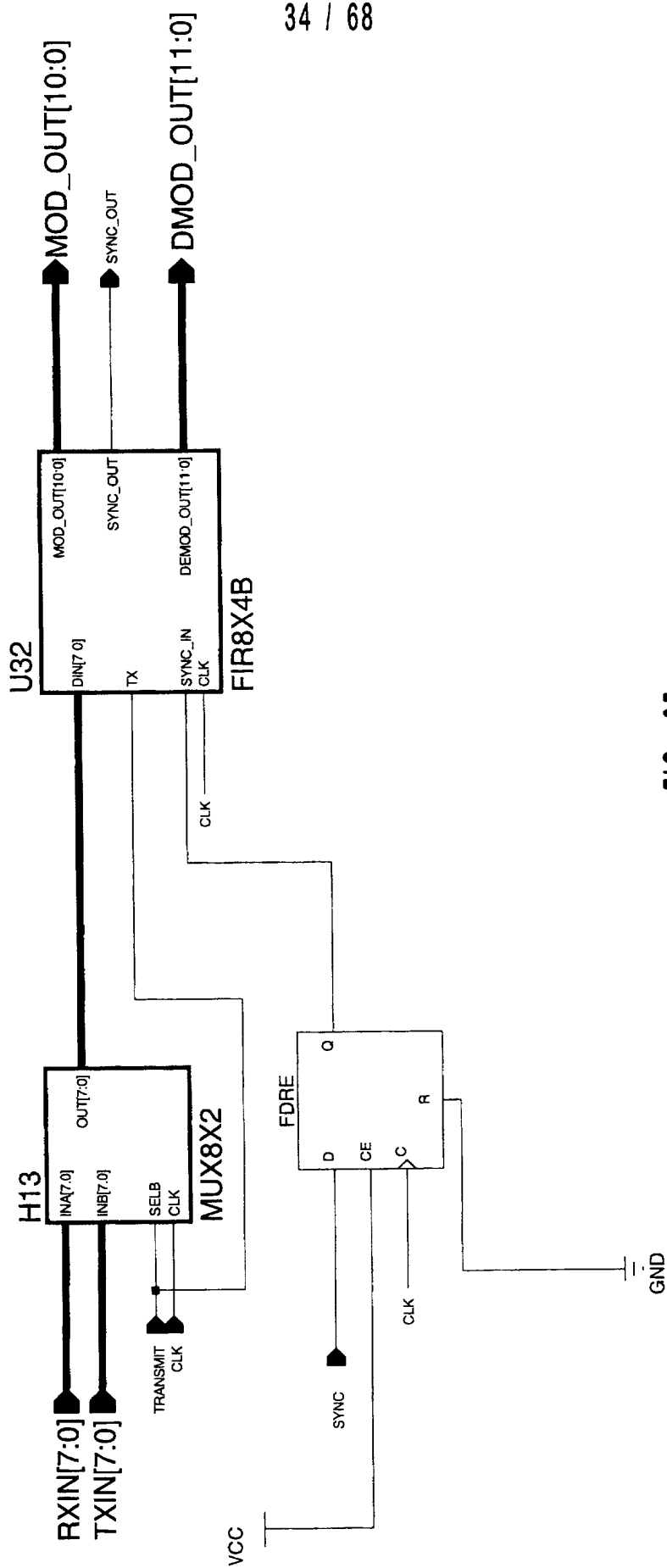


FIG. 35e

FIG. 35f-1

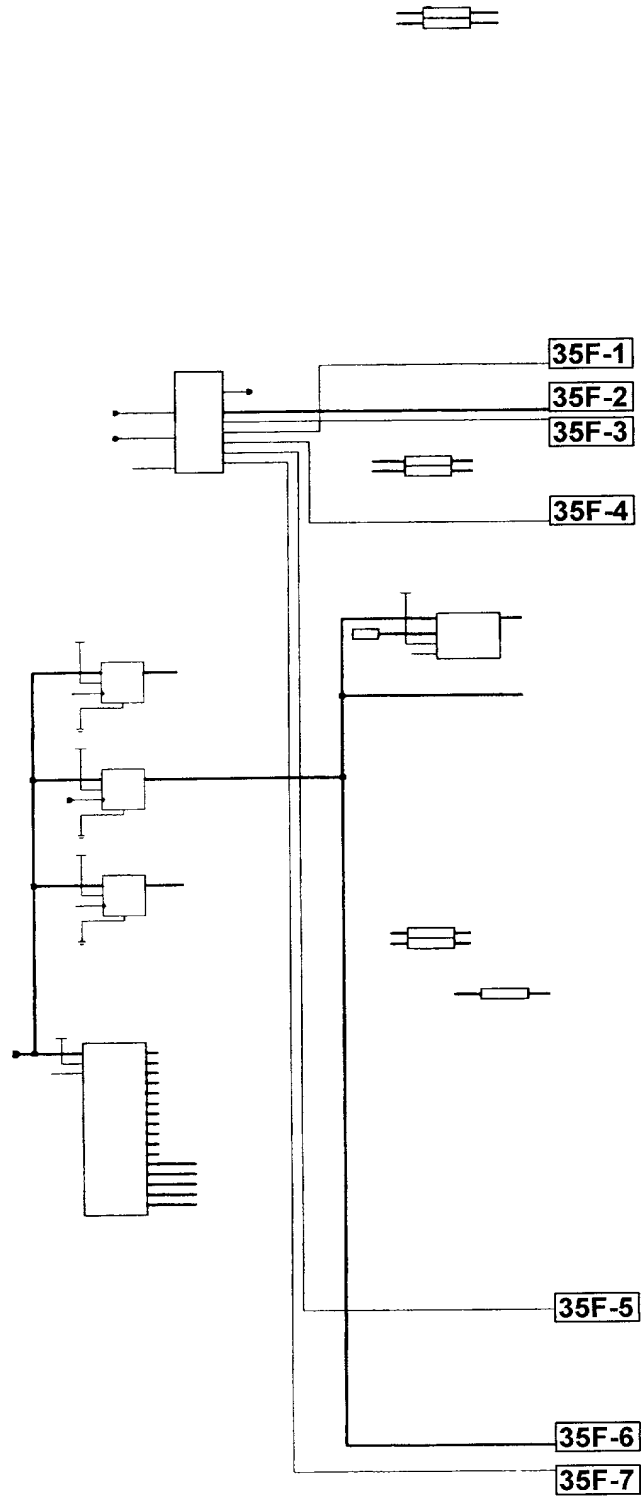


FIG. 35f-1

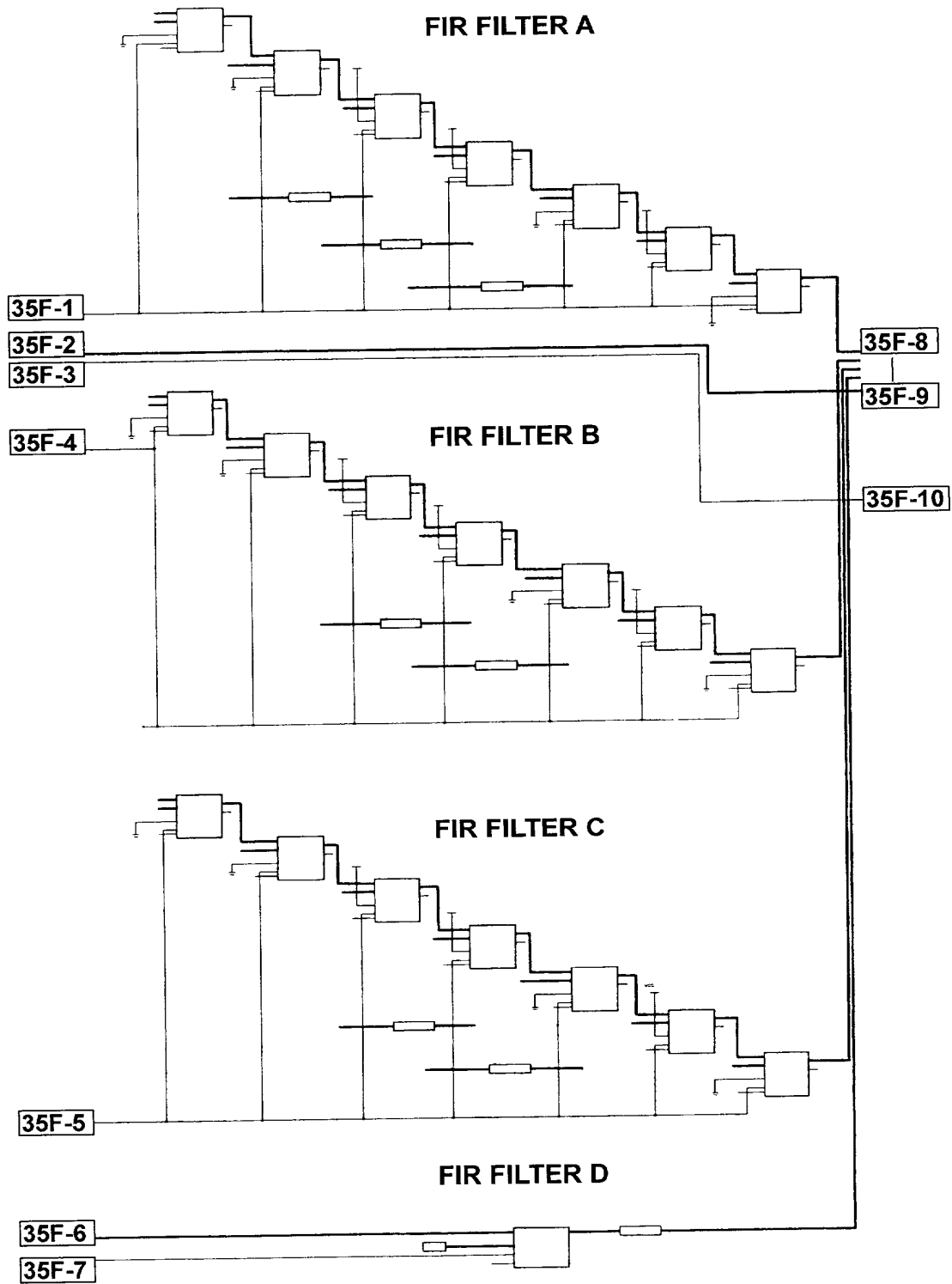


FIG. 35f-2

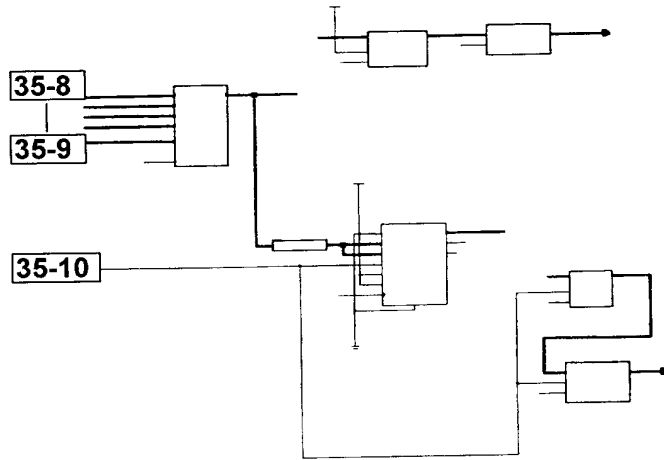


FIG. 35f-3

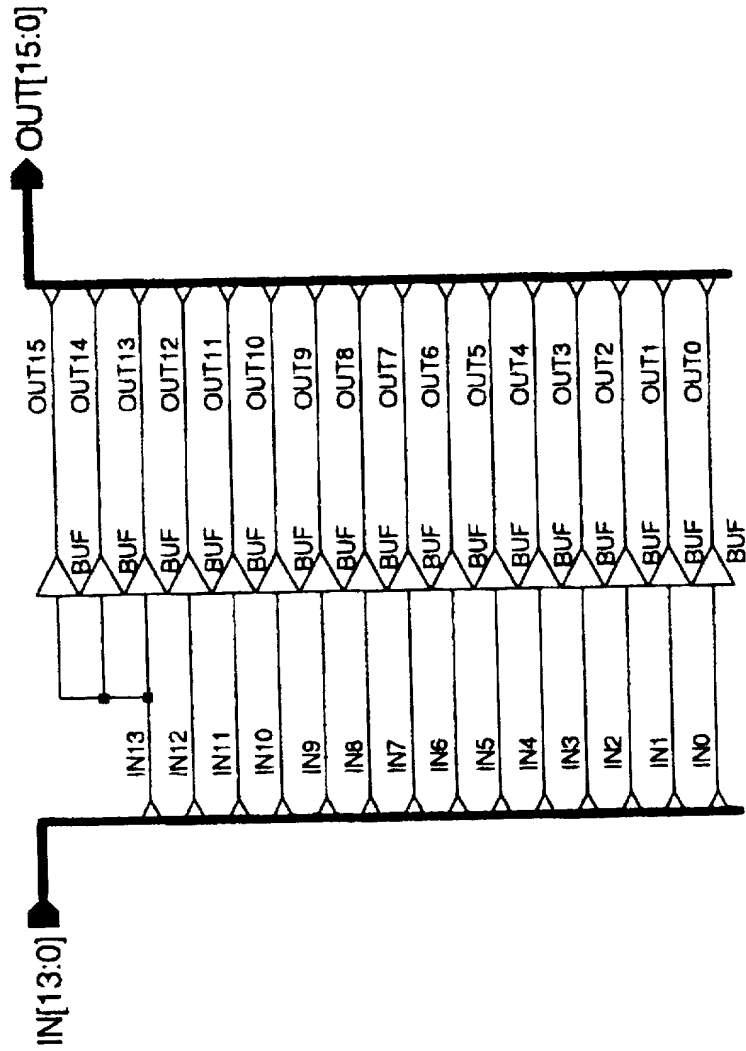


FIG. 35g

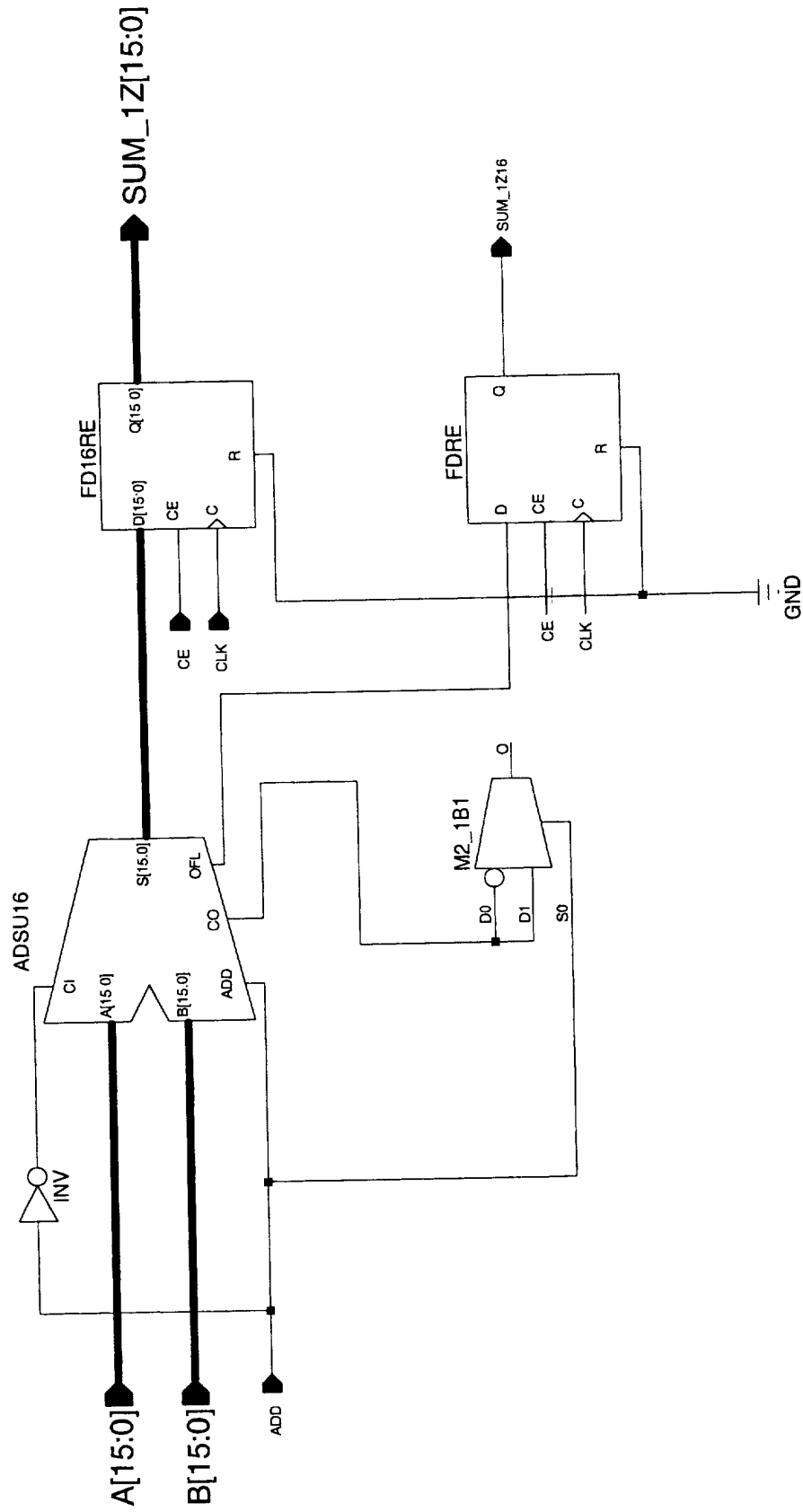


FIG. 35h

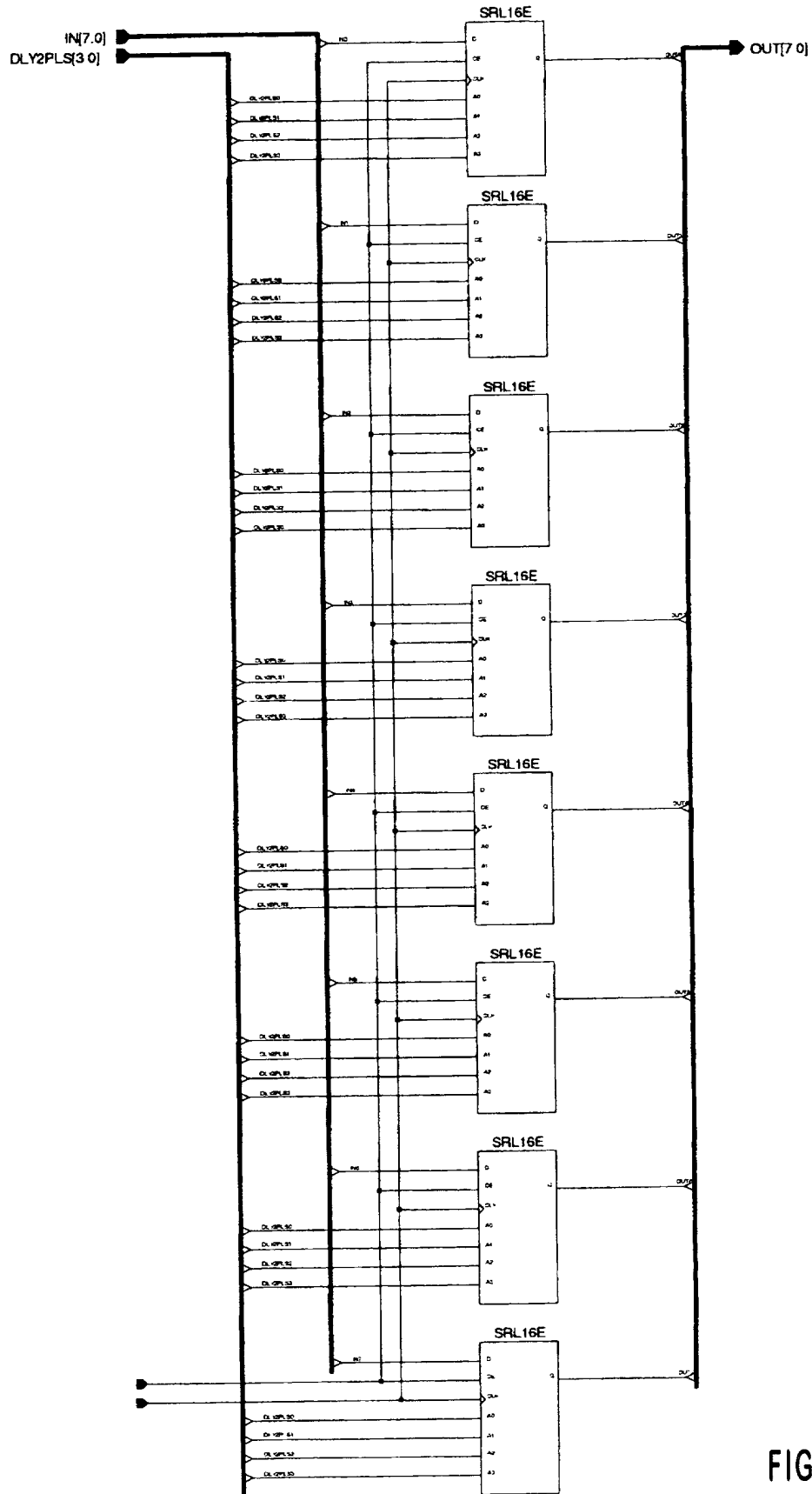


FIG. 35i

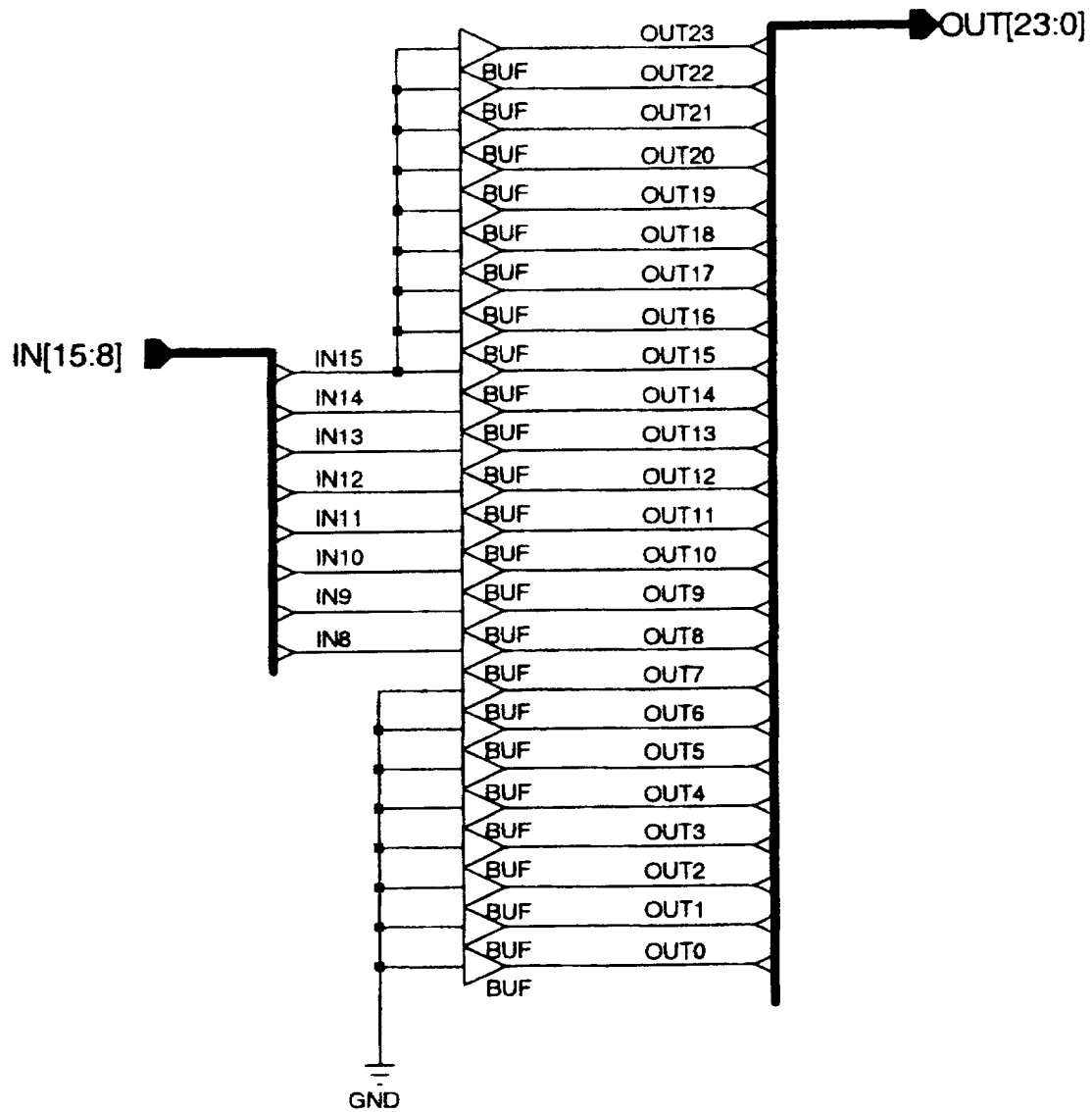


FIG. 35j

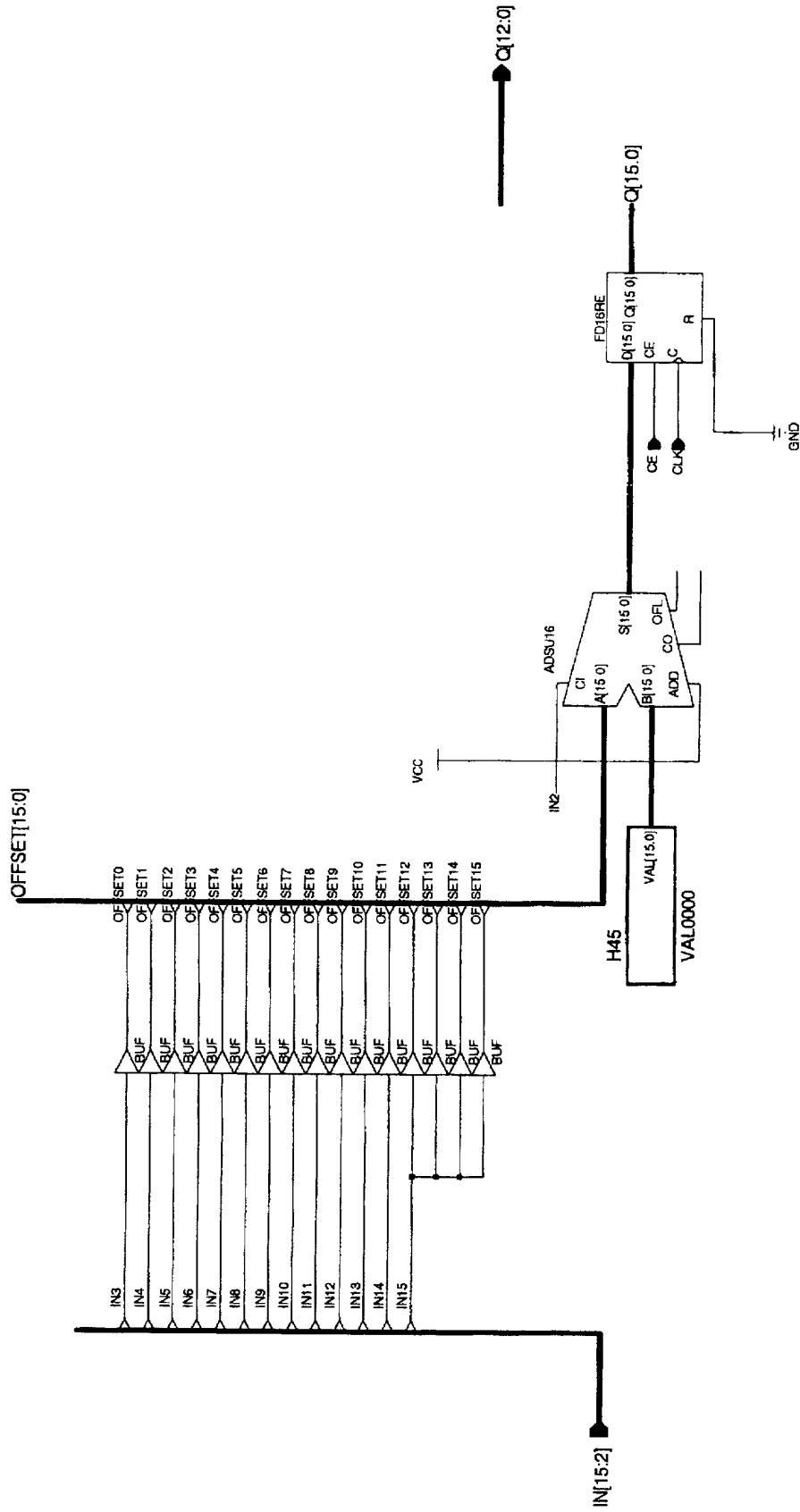


FIG. 35k

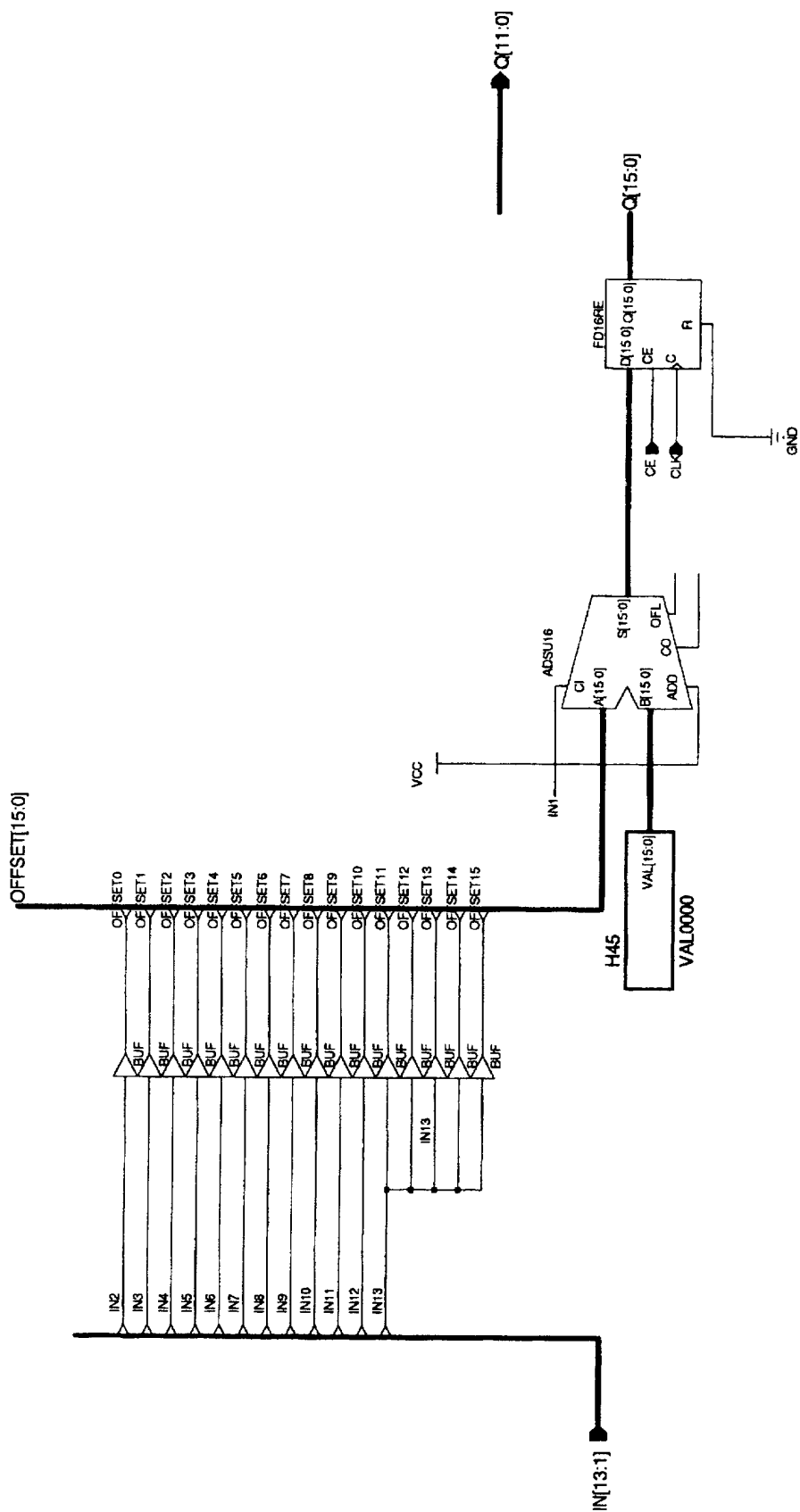


FIG. 35I



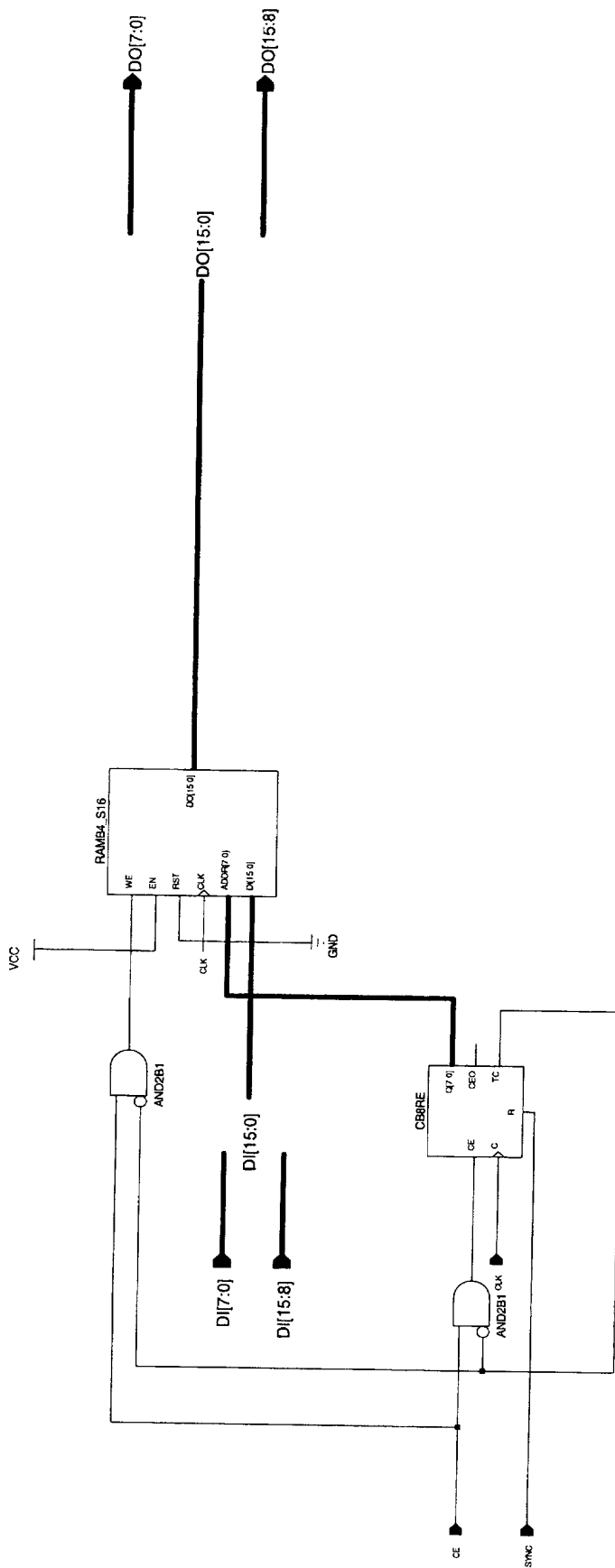


FIG. 35n

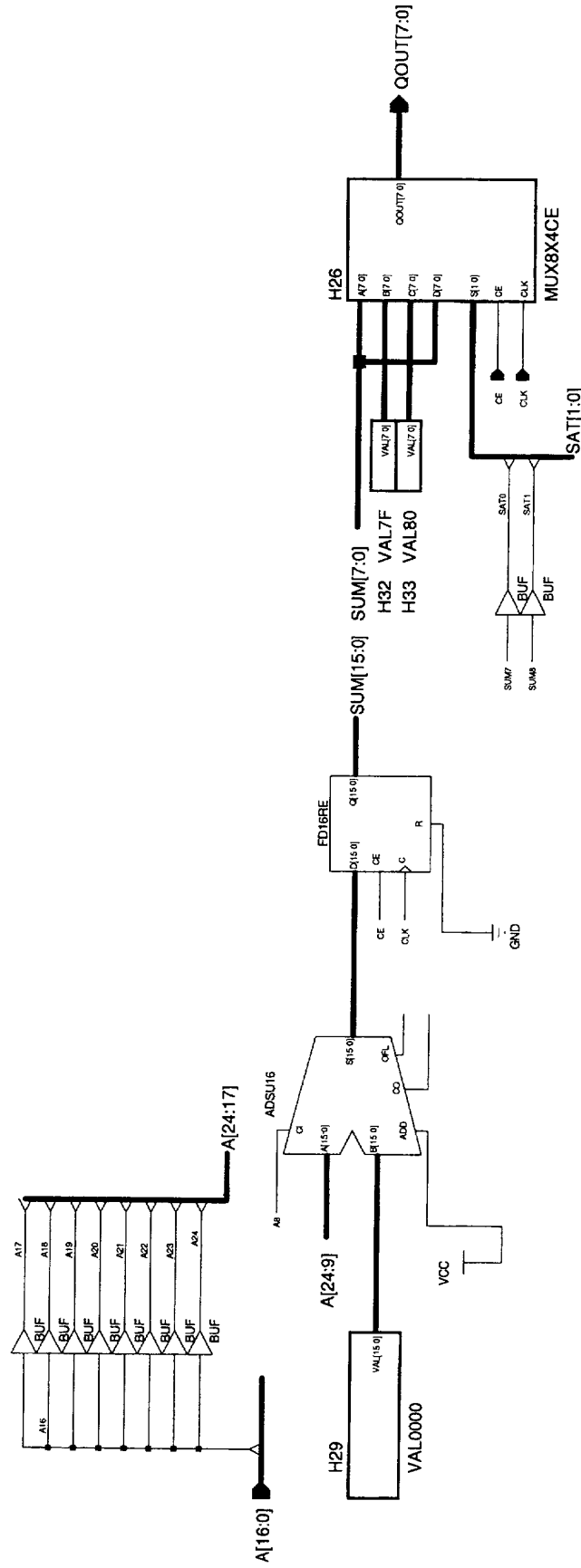


FIG. 350

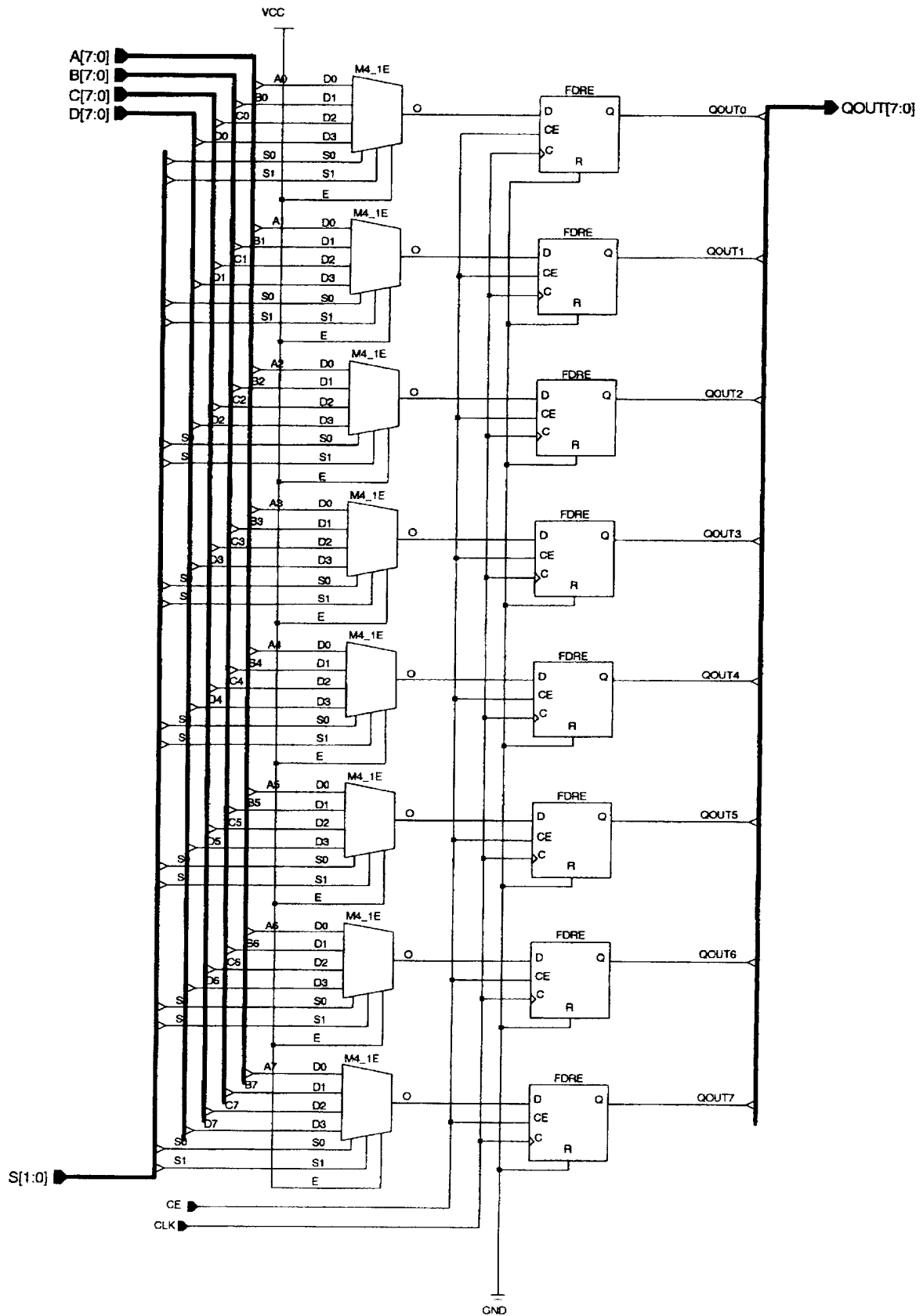


FIG. 35p

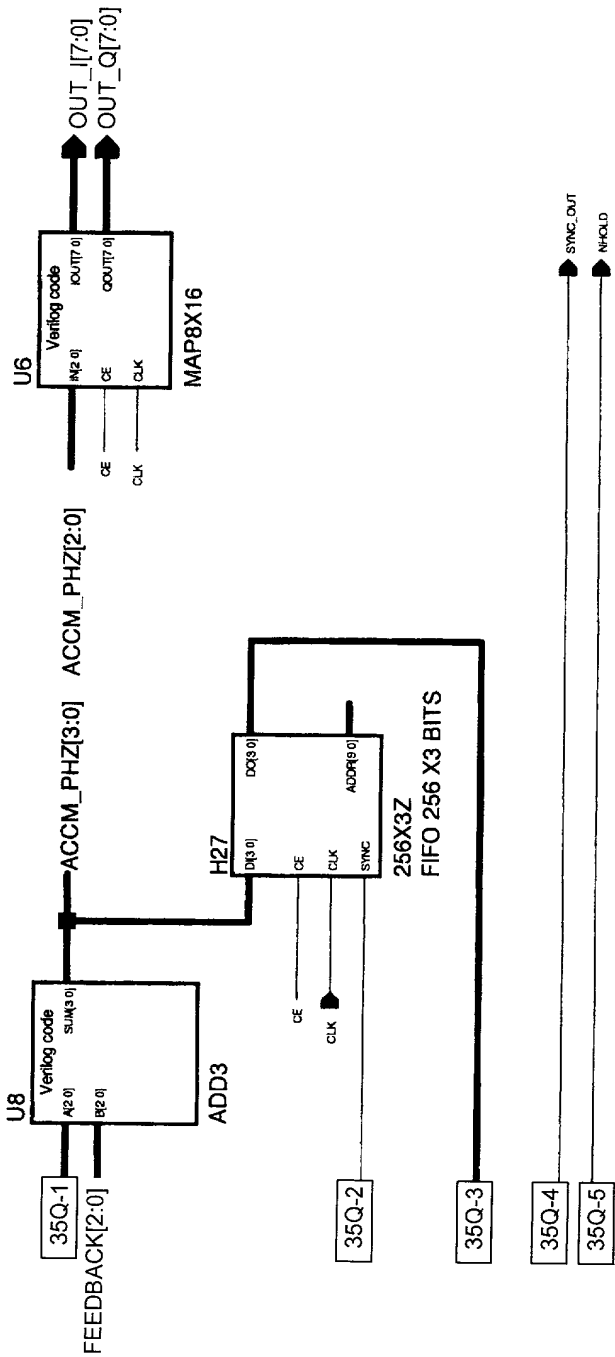


FIG. 35q-2

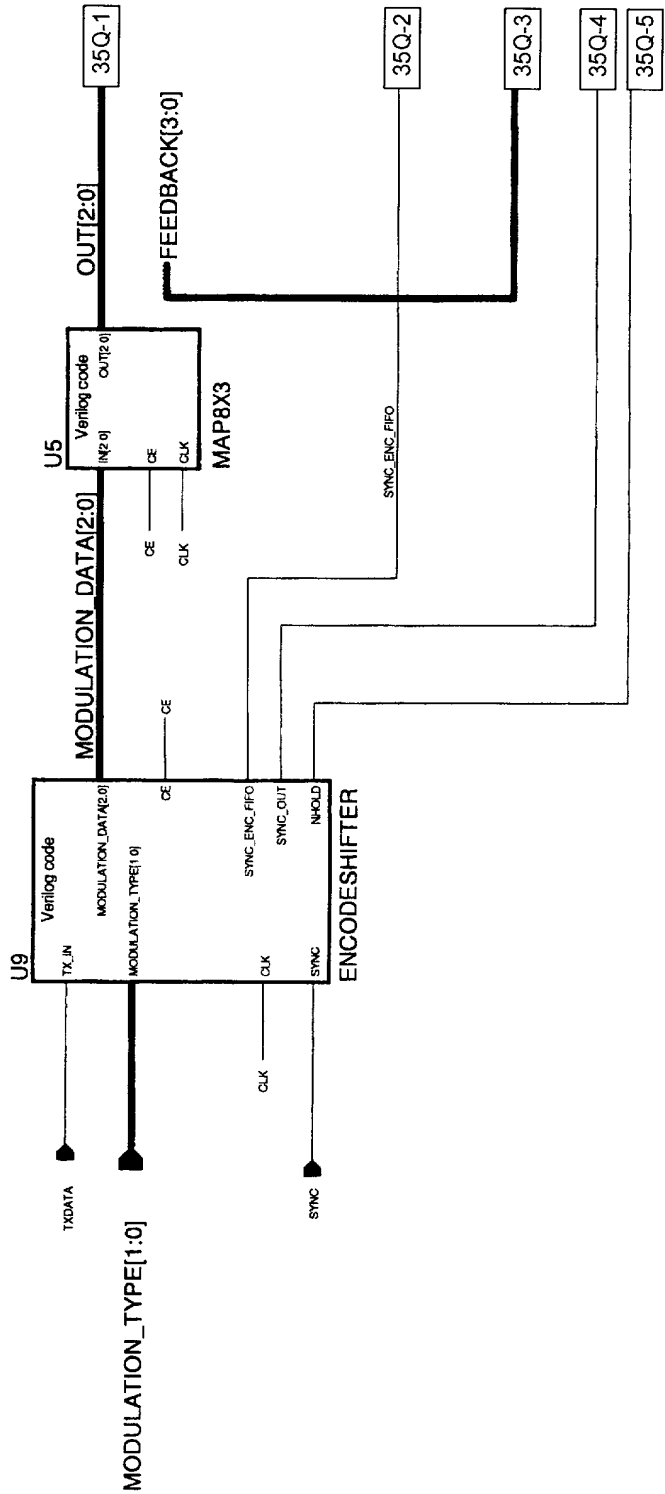


FIG. 35q-1

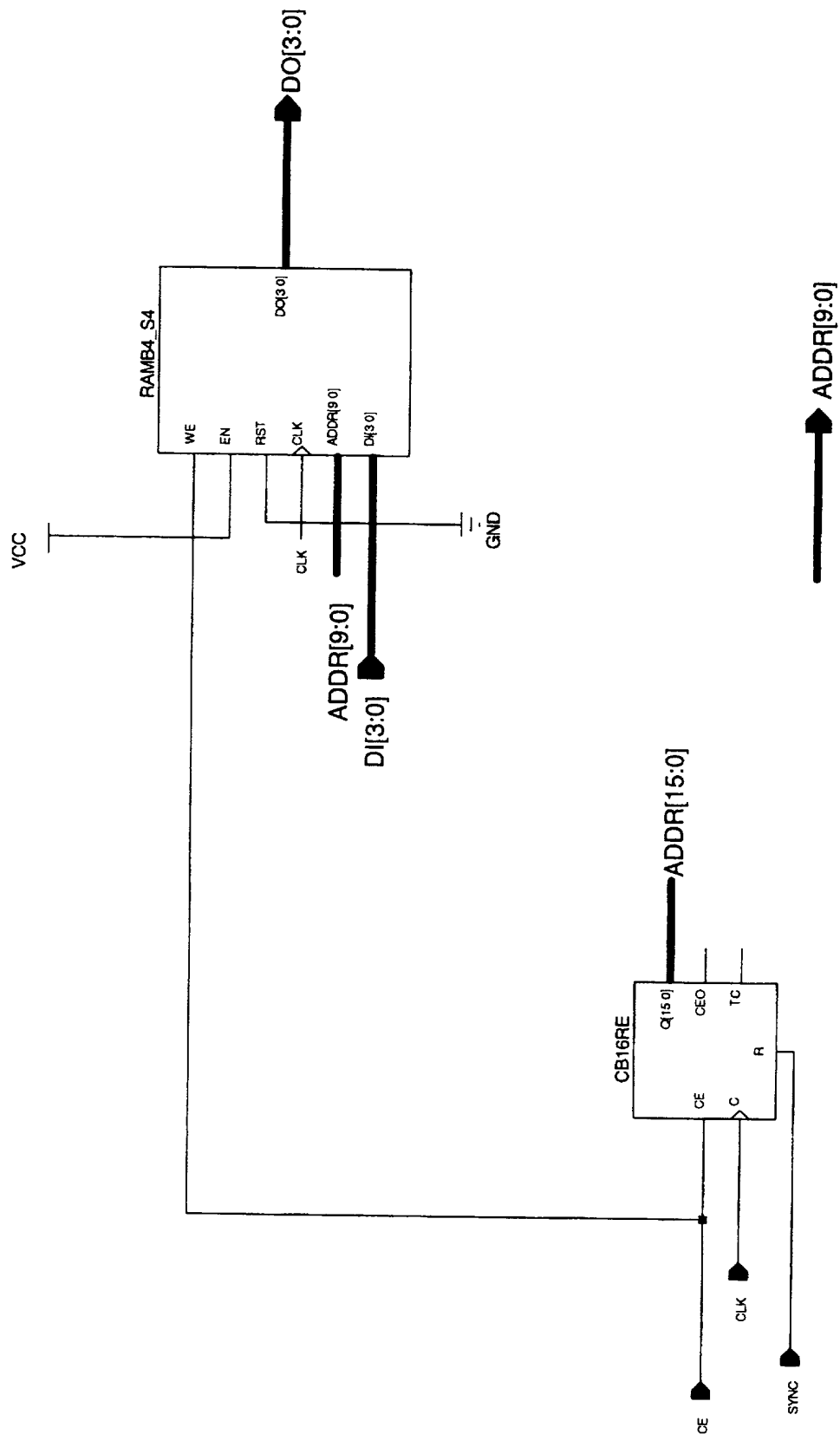


FIG. 35r

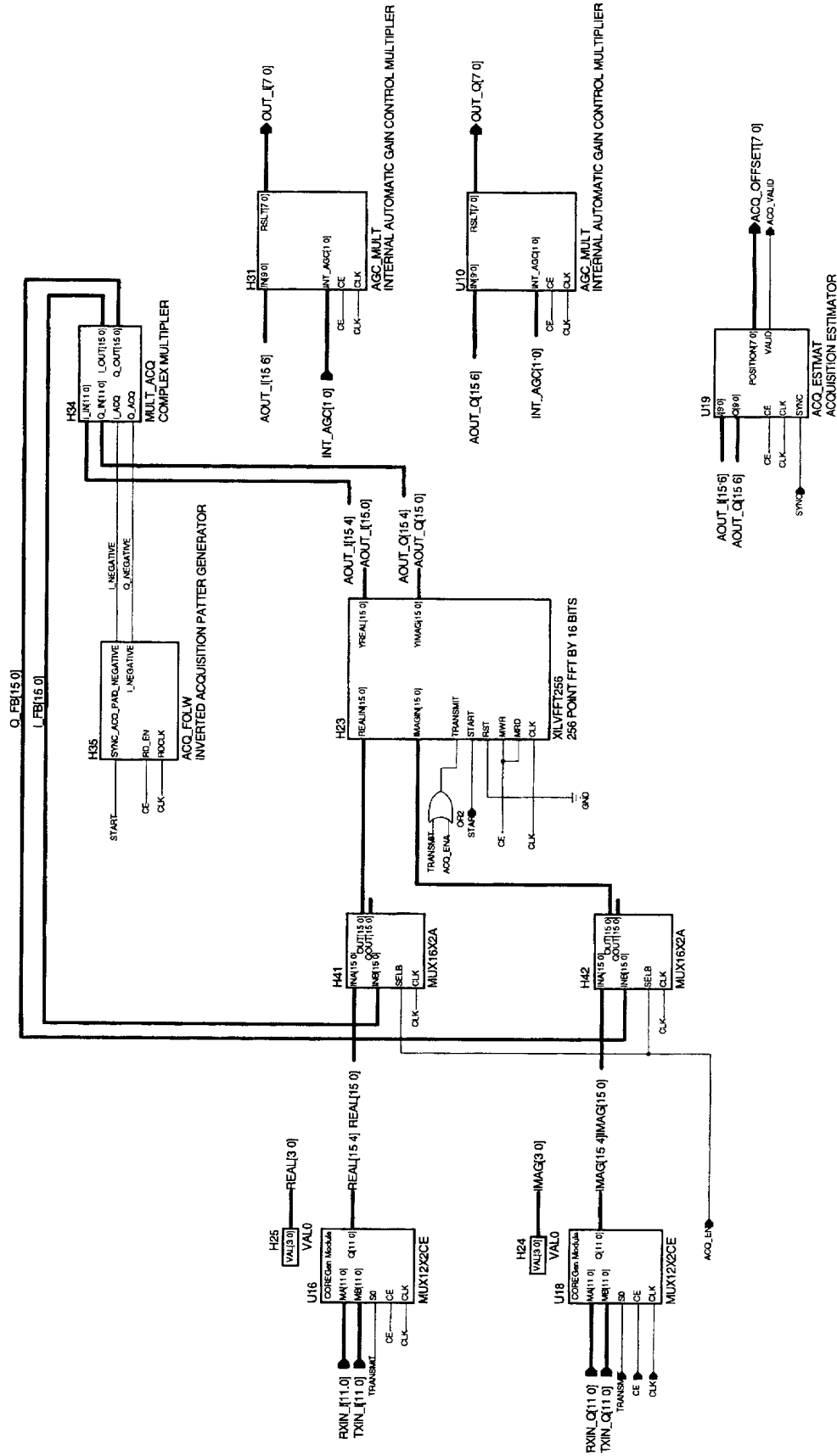


FIG. 35s

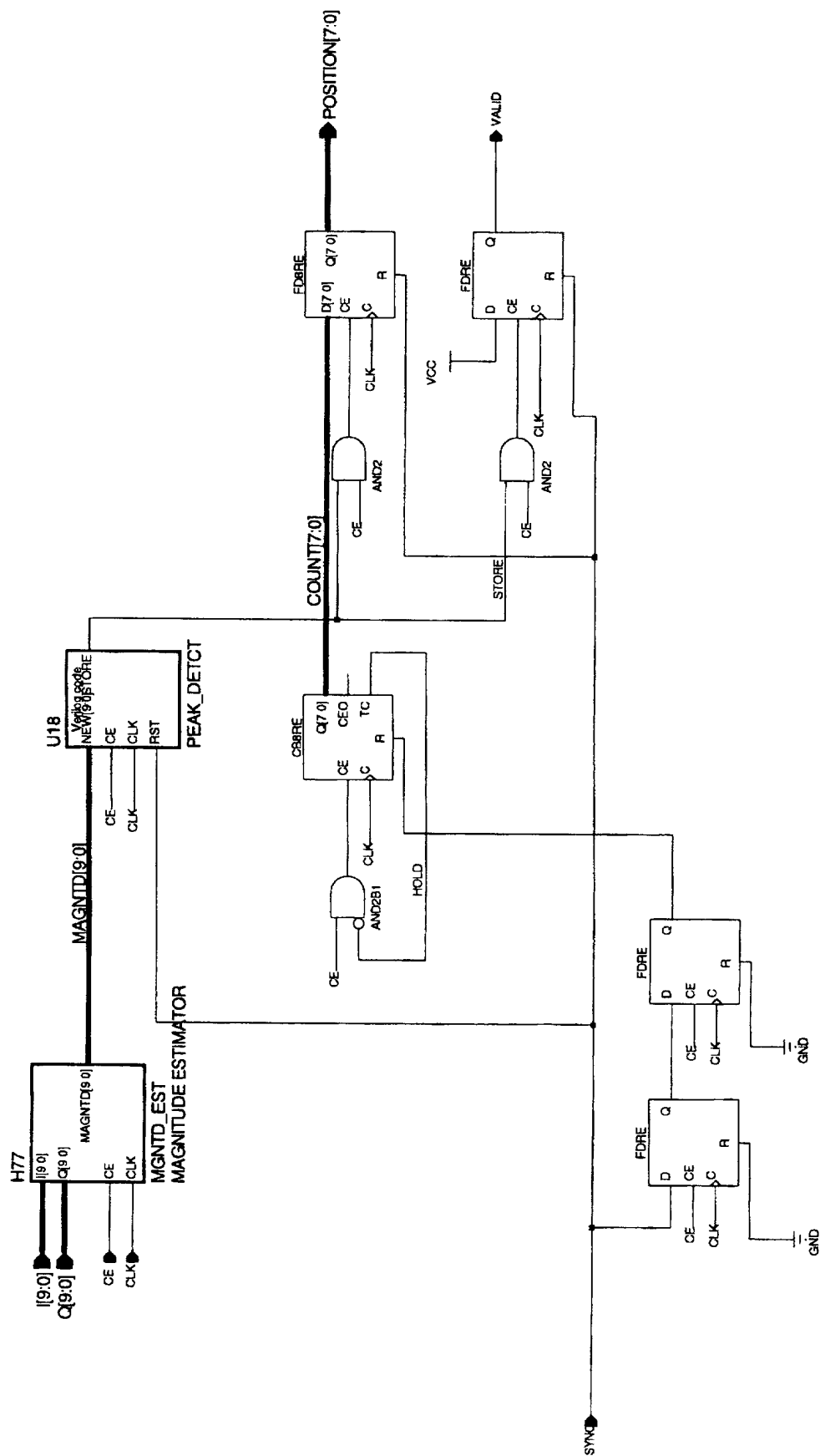


FIG. 35t

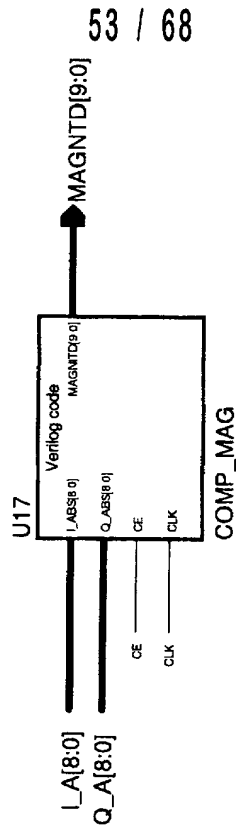
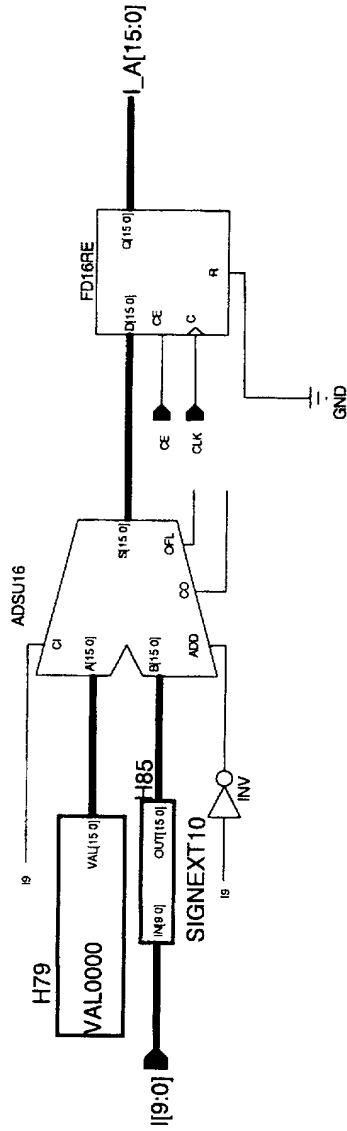


FIG. 35u

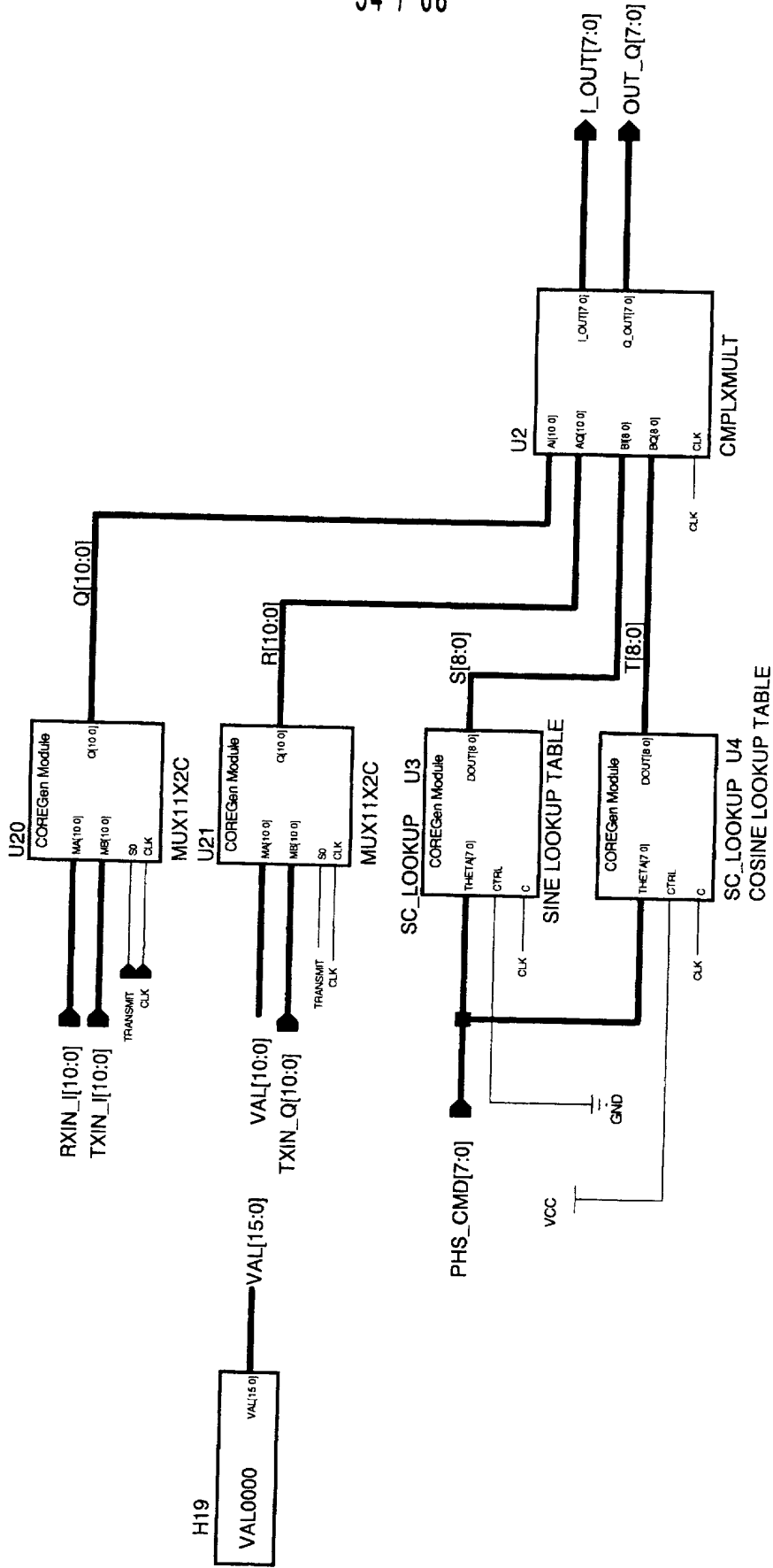


FIG. 35V

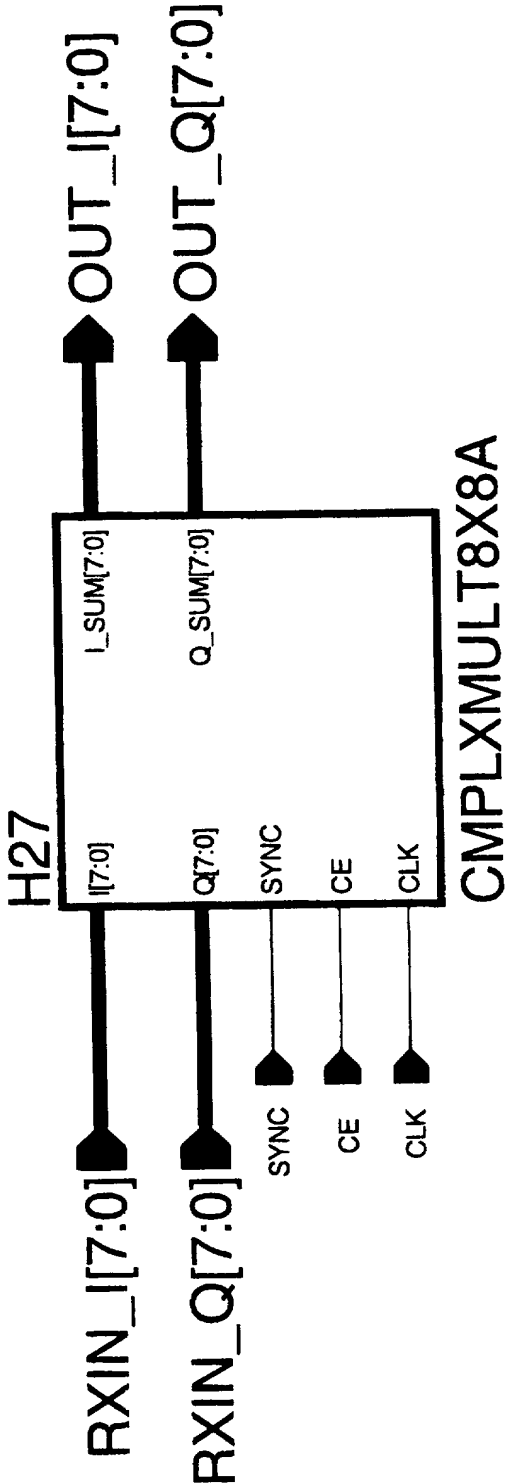
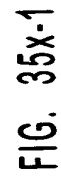


FIG. 35W



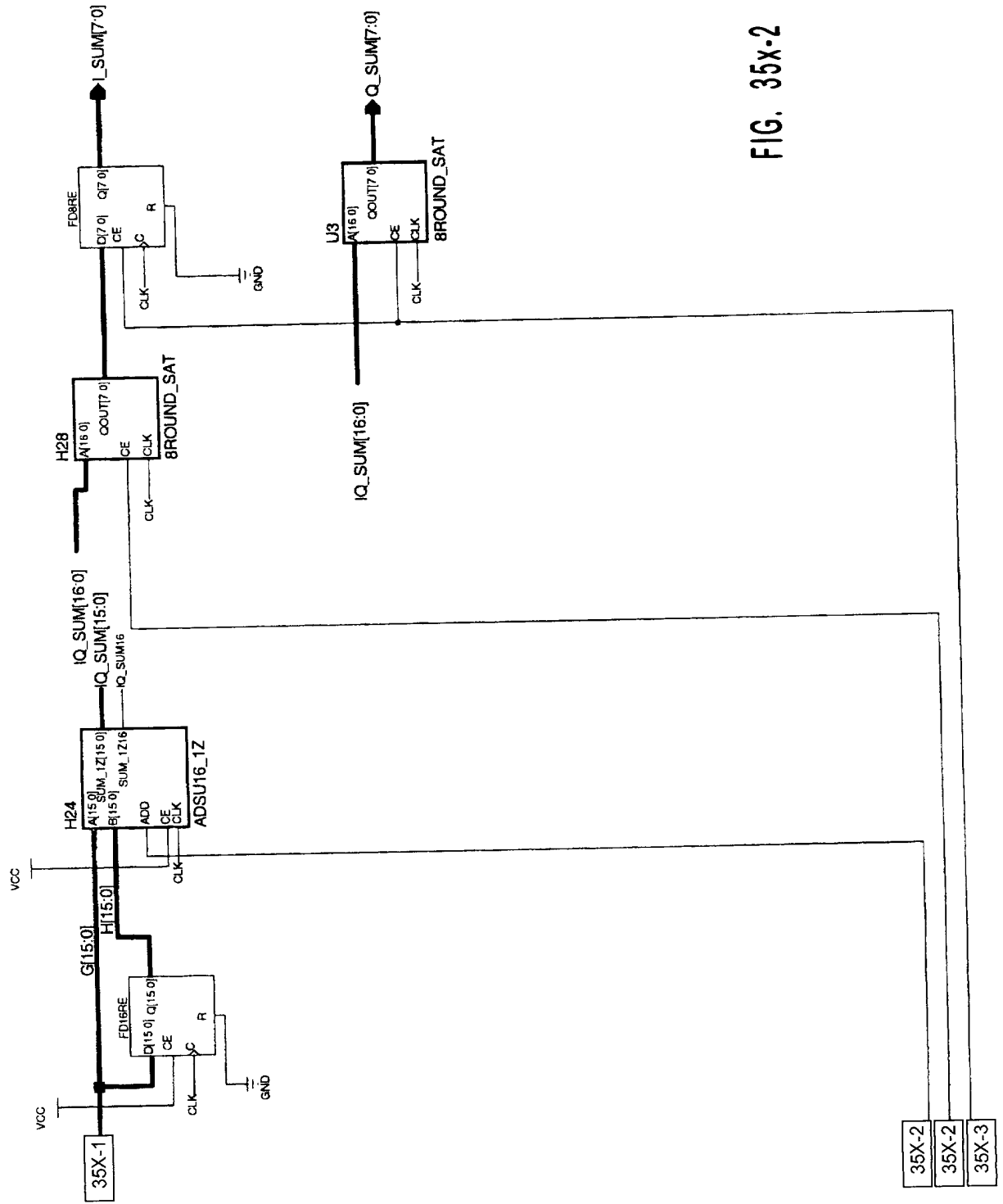


FIG. 35X-2

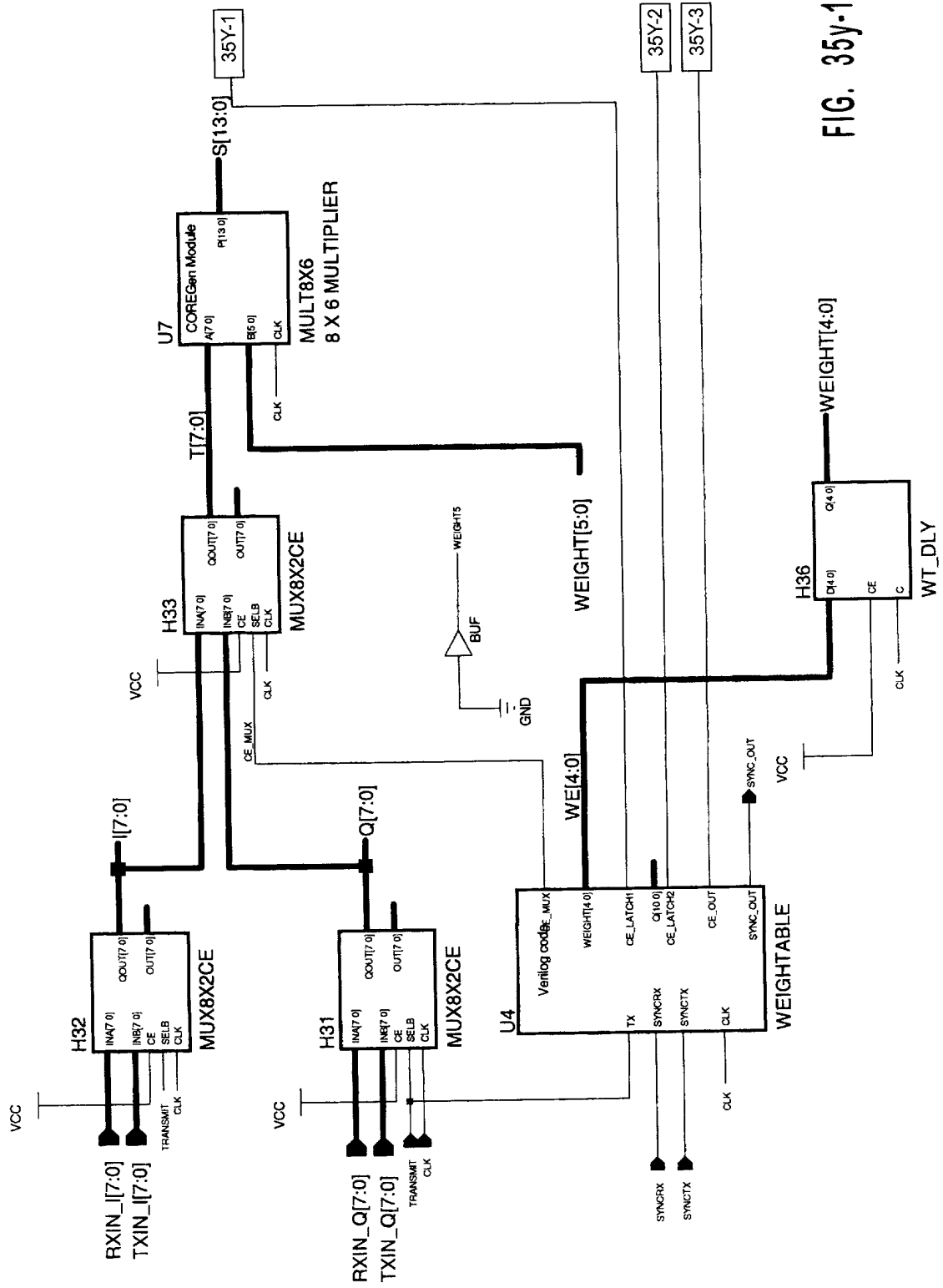


FIG. 35y-1

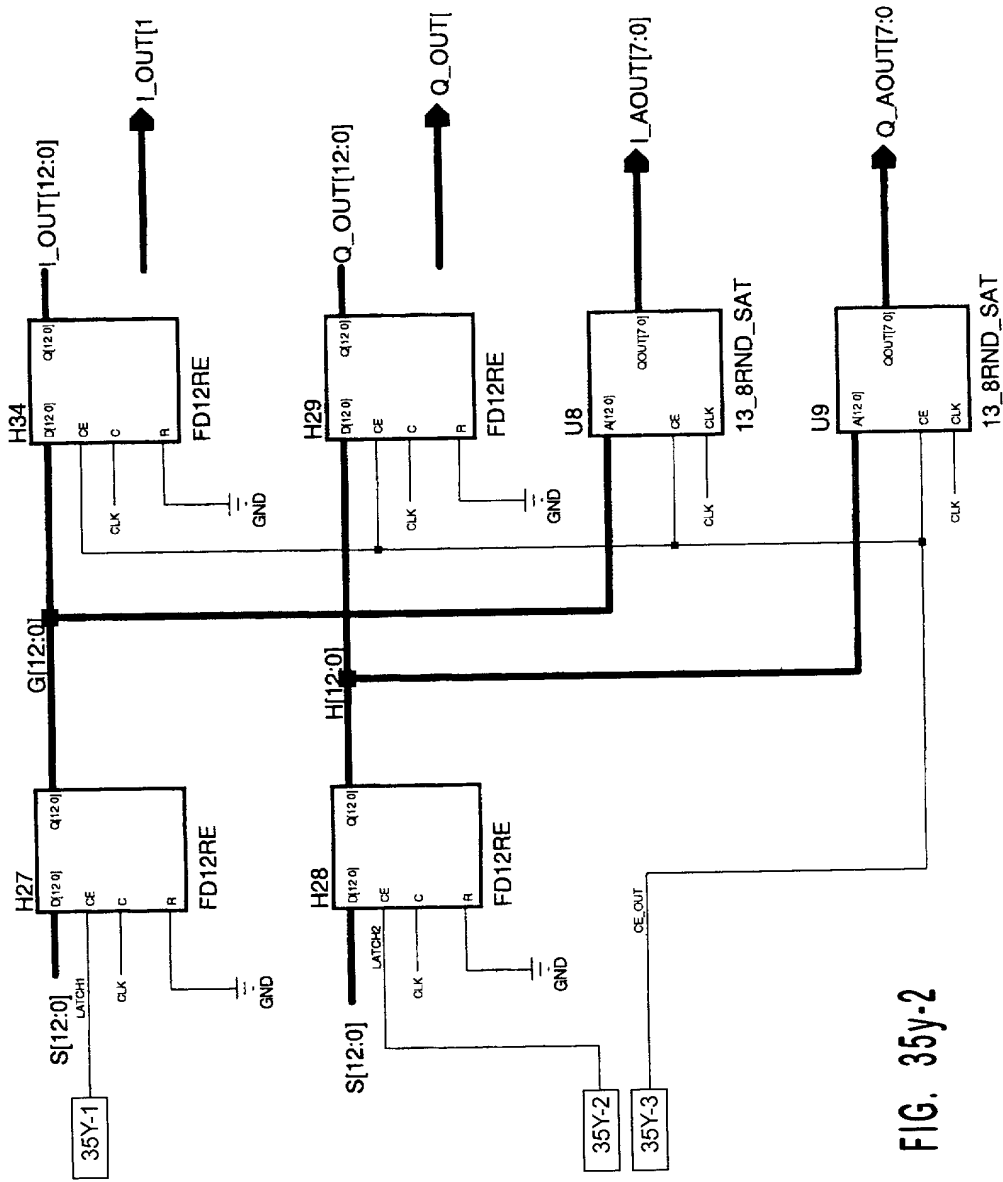


FIG. 35y-2

FIG. 35Z

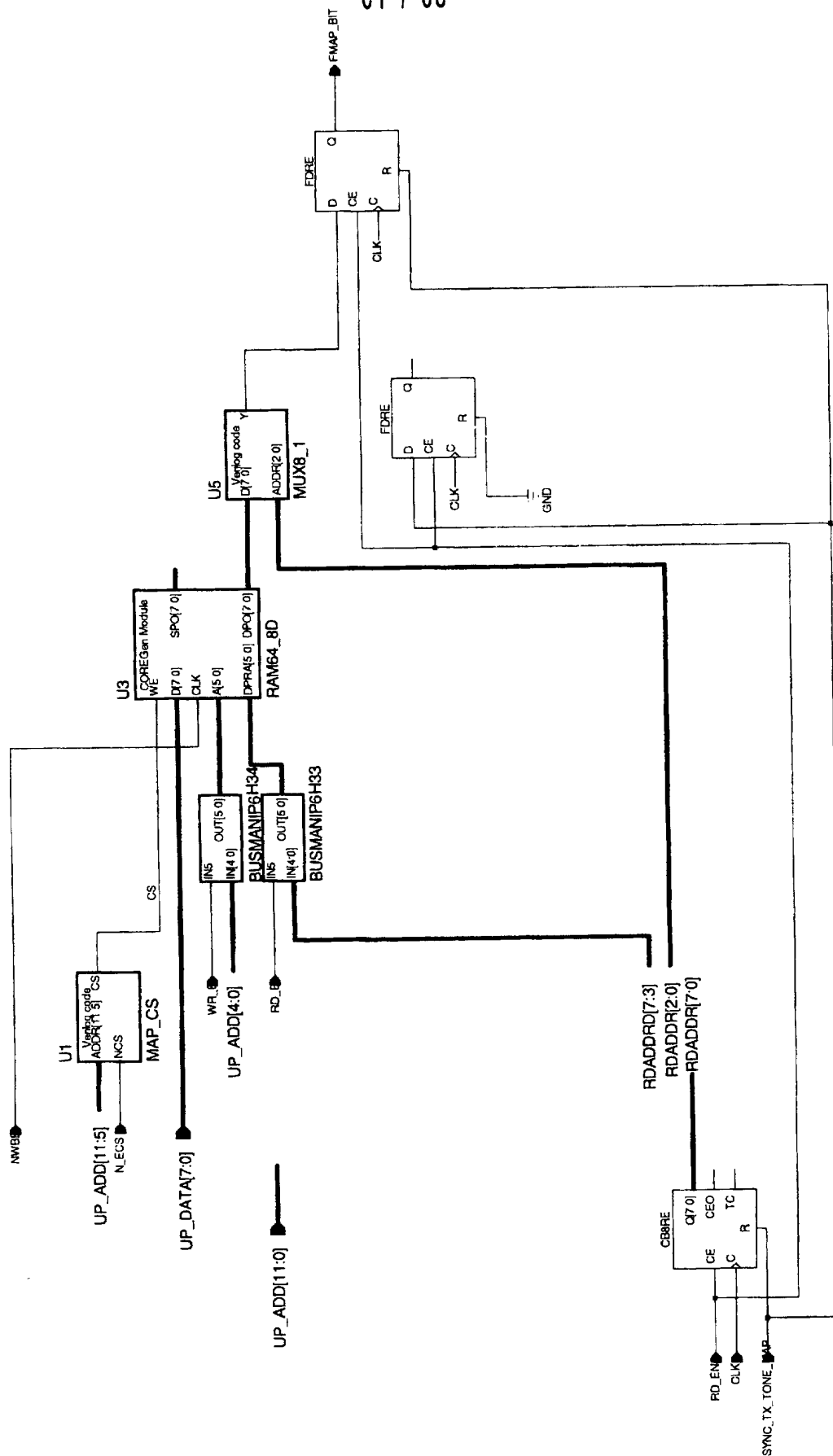


FIG. 35aa

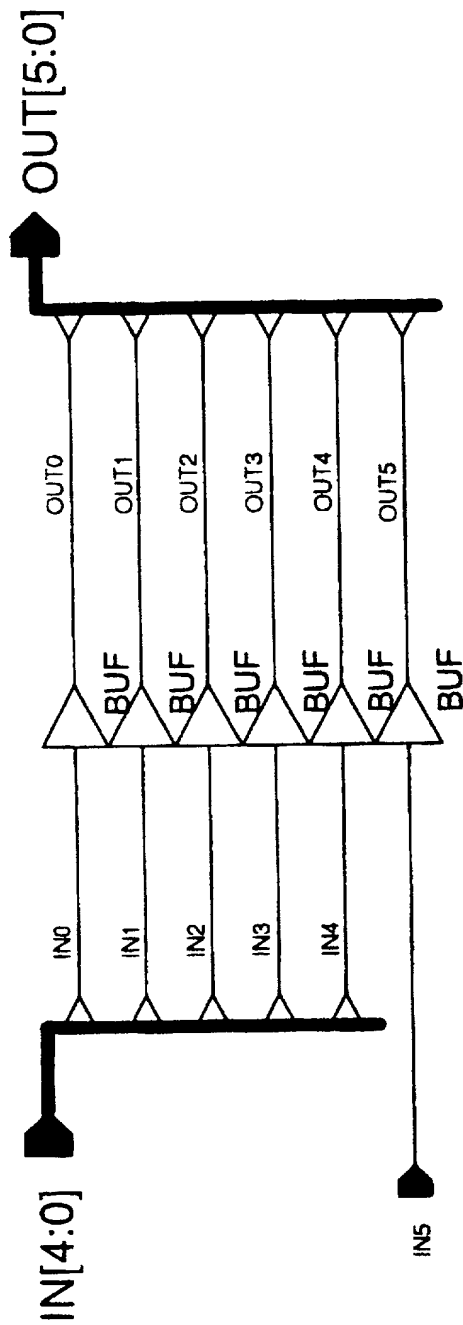


FIG. 35ab

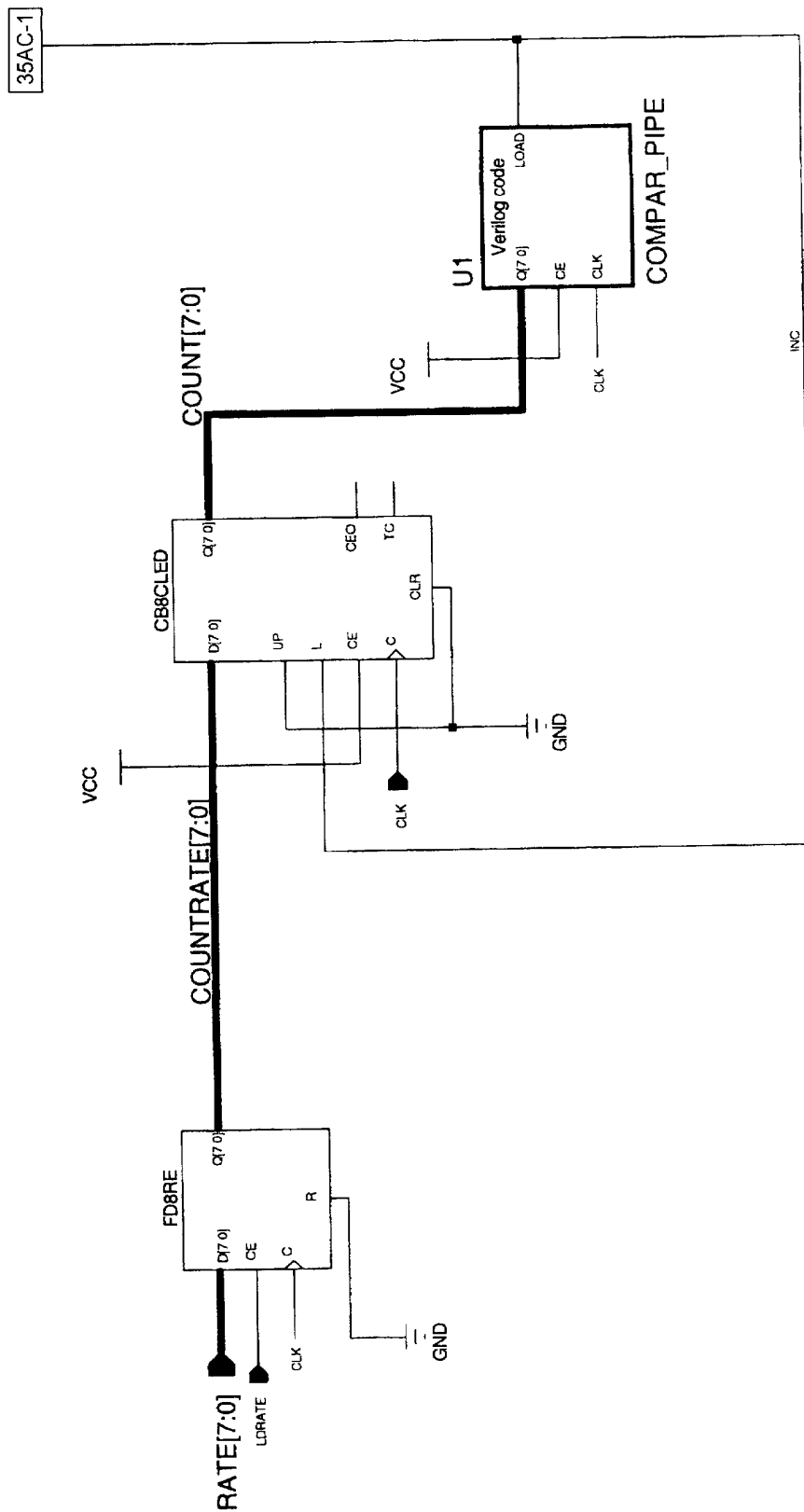


FIG. 35ac-1

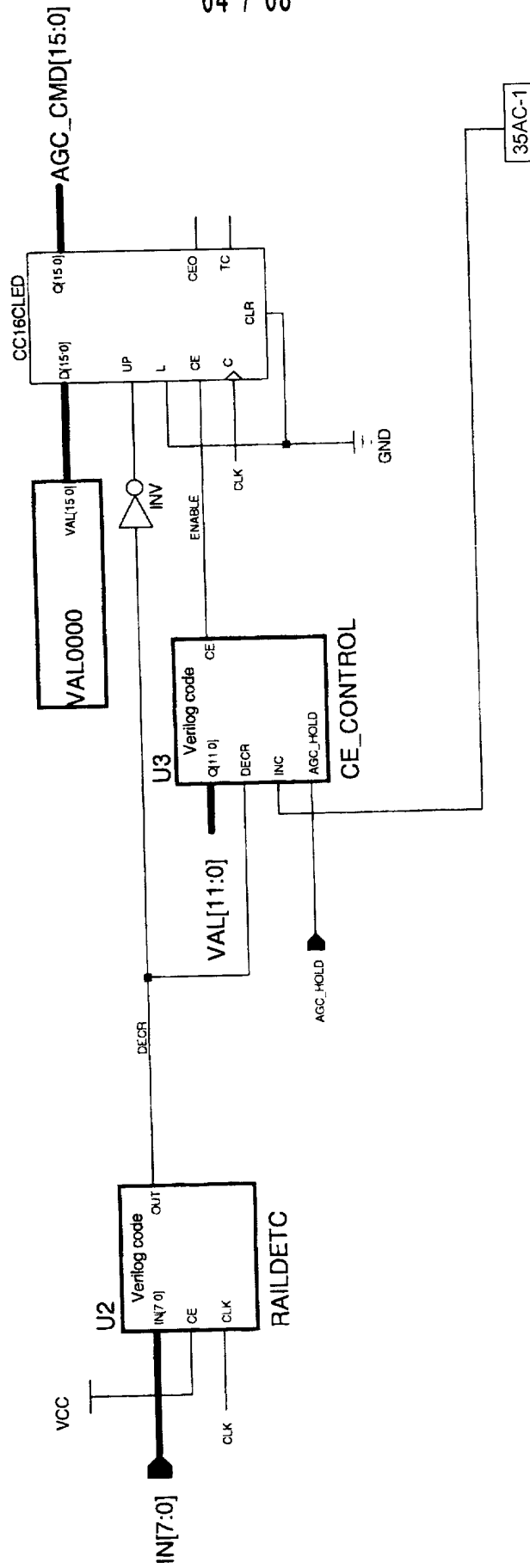


FIG. 35ac-2

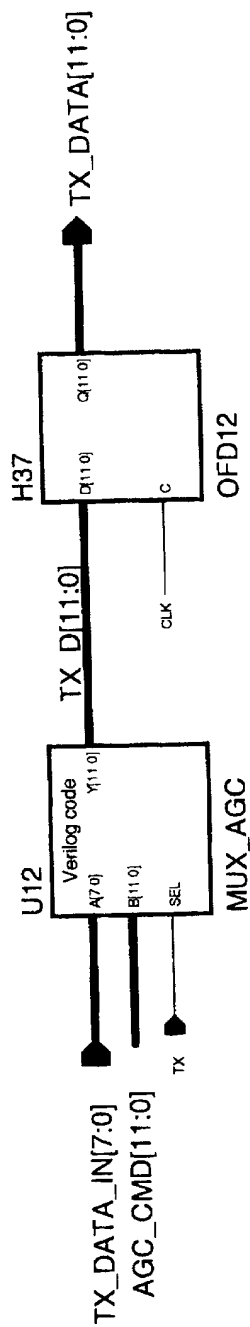


FIG. 35ac-3

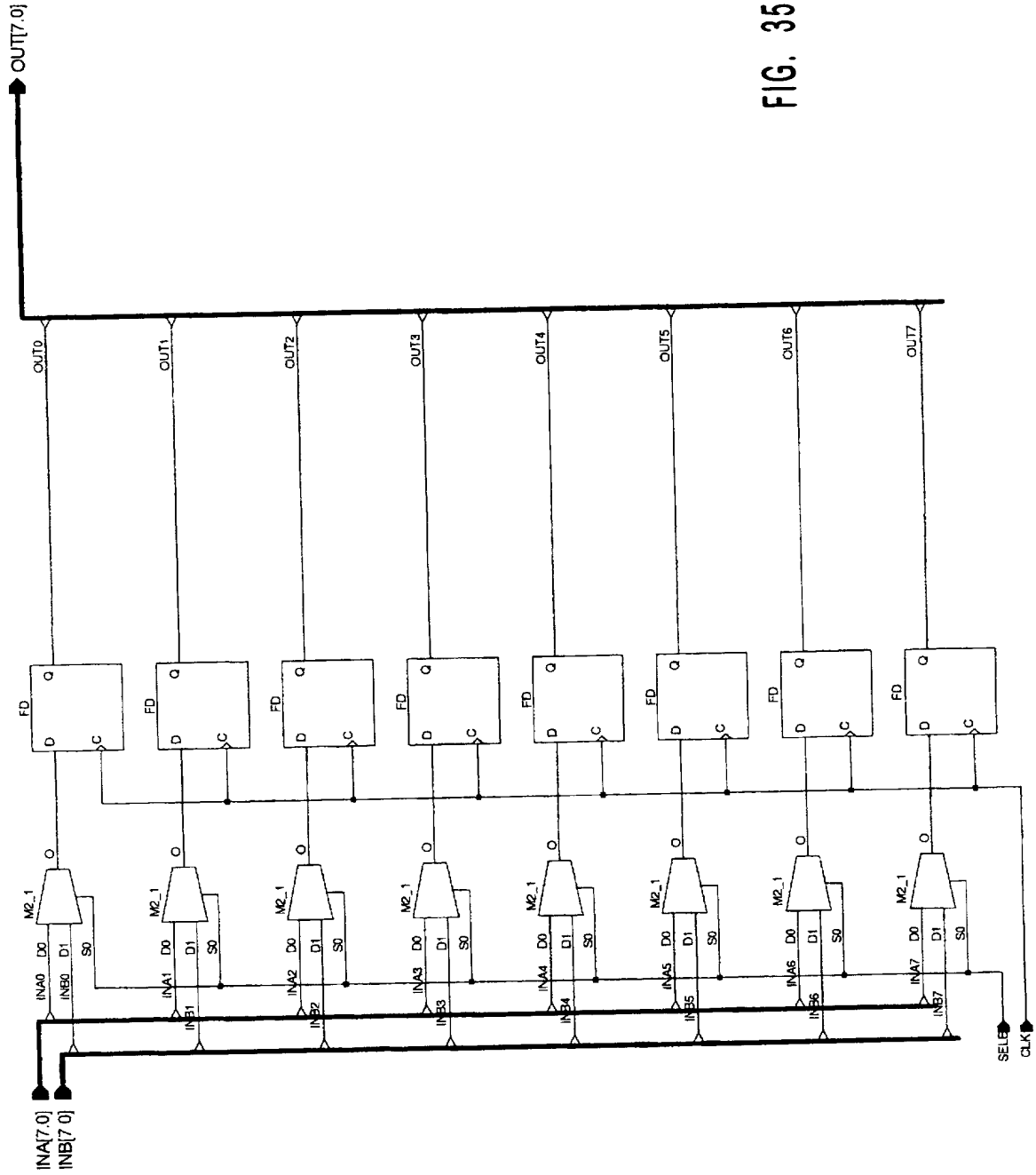


FIG. 35ad

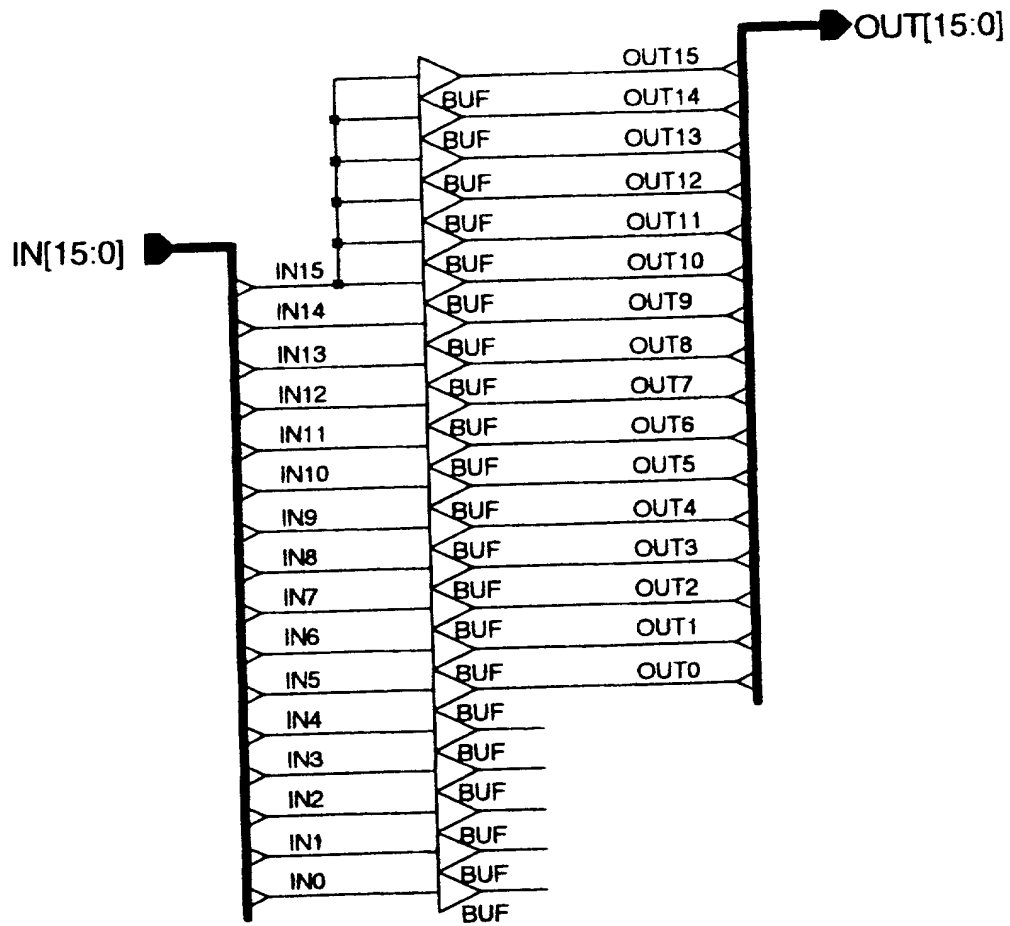


FIG. 35ae

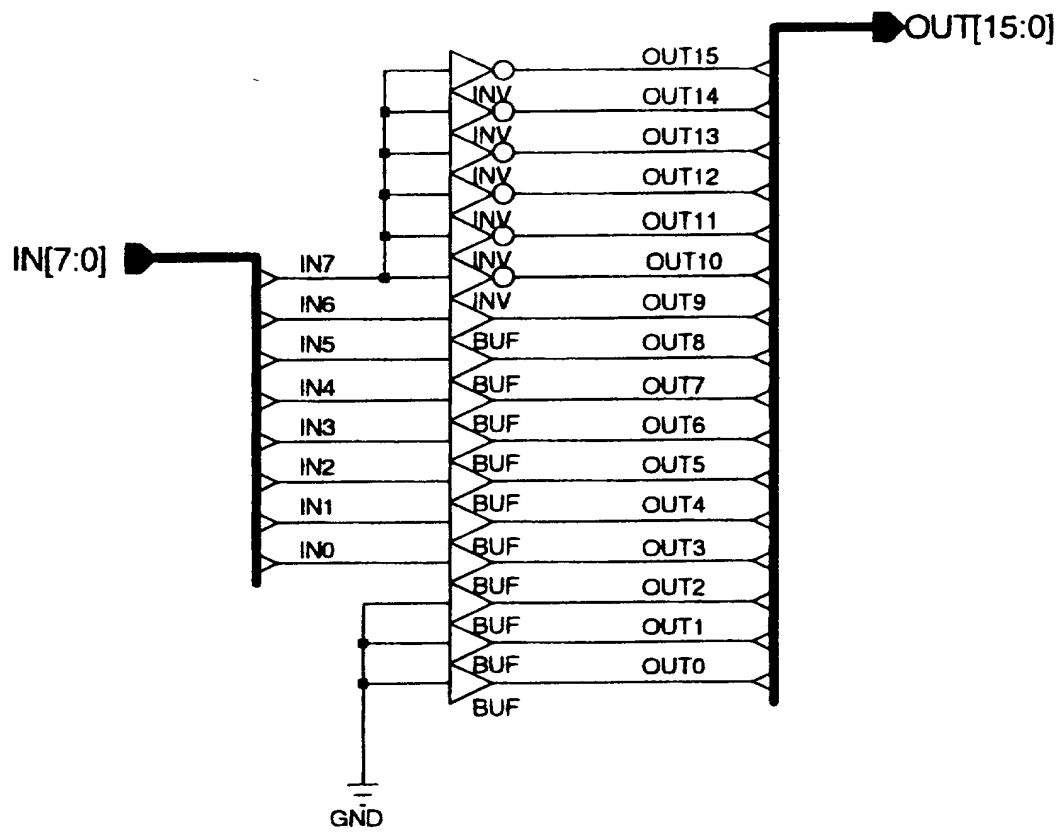


FIG. 35af